

## Optimization process of a multi-reservoir system in a market context

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### Abstract

This paper presents the process used by ENEL in order to optimize the tasks of scheduling, controlling and operating of multi-reservoir systems within the Italian energy market. The process described below is divided in three steps: definition of the constraints, real time operation and real time actuation. On the whole, the process covers a timeframe that begins up to 14 days before the day of delivery and ends with real-time actuation. Each step is characterized by a proper optimization process with different description of the objective function, constraints and with a different focus on the time horizon to be evaluated.

### Introduction

From the point of view of hydroelectric power production, the introduction of the market context has completely changed the way of exploitation of reservoir systems. Before the market liberalization, the prime objective was to perform the operating policy aiming the lowest use of water by avoiding spilling and by maximizing the hydropower generation, as well as satisfying all operating constraints. On the contrary, in the market context the focus is to obtain the maximum of the economical income respecting not only the operating constraints, but also the market mandatory.

Table 1 summarizes the tools, designed and utilized by ENEL and implemented by CESI, for the different steps of the optimization process: **OMA** (Italian acronym for several days ahead optimization in a market framework), **OASI** (real time optimization of a multi-reservoir system), **OPG** (real time optimization of the actuation).

Table 1: Step and tools in the optimization process

Step	Tool	Time horizon	Time step
Definition of the constraints	OMA	Several days	1 hour
Real time operation	OASI	1 day	15 minutes
Real time actuation	OPG	6 hours	1 minute

### Definition of the constraints

#### Generalities

This step of the process is performed several days before (up to 14 days) the day of delivery (D). The tool used is OMA and its aim is **not the optimization of the hydro power plants' production**, but **the definition of the operational limits**, which must be considered during the bidding phase.

The definition of the constraints is performed at the regional centers; each one is responsible for managing several independent multi-reservoir systems.

The main inputs of OMA are the seasonal definition of the desired levels for each reservoir at the end of the given time steps (i.e. at the end of a week), the planned maintenance for power plants, the environmental constraints, other field measurements such as the stored water level and the past natural flows. The latter lets the evaluation of the future natural flows, one of the most important OMA's input, for each reservoirs, in each time step, as it's described in [1].

Given the above mentioned constraints, it is possible to have an infinite number of non-optimal physical solutions for the dispatching: **the purpose of OMA is to find the upper and lower envelopments of all the possible physical solutions**. These bounds are then used during the bidding phase, which is performed at the national control center and where the multi-reservoir systems are considered with a lower physical detail.

To find all the existing solutions for every reservoir systems, OMA works in two steps. The **first is the check of the energy market schedules** for D, so to predict final volumes and discharges. With the above-mentioned results, the **second** step allows, throughout the course of several optimizations, to **ascertain the operational limits** from D+1 up to 14 days ahead.

#### General problem statement

In the adopted model, the optimization horizon is a set named HOURS, which contains all the hours from the

start to 24-th hour of the last day ahead to optimize; moreover PU and RESERVOIRS are respectively production units and reservoirs sets. Other considered sets are: Der, including all the derivations of multi-reservoir system (power-pipes starting with a reservoir and ending to unit), Cha, containing the list of channels (starting from power plant and ending into a reservoir), Pen, including the list of penstocks (channels starting from head water and ending to collecting reservoir) and Spill, that is spillways' set (channels starting from head water and ending to tail water with emergency function).

It gives rise to the following **linear programming constraints**.

The hydraulic balance of all the reservoirs for each hour:

$$\begin{aligned} V_{i,h} = & V_{i,h-1} - \sum_{(i \rightarrow u, tc) \in Der} Qder_{(i \rightarrow u, tc), h} + \\ & \sum_{(u \rightarrow i, tc) \in Cha} Qcha_{(u \rightarrow i, tc), h-tc} - \sum_{(i \rightarrow j, tc) \in Pen} Qopen_{(i \rightarrow j, tc), h} + \\ & \sum_{(j \rightarrow i, tc) \in Pen} Qopen_{(j \rightarrow i, tc), h-tc} - \sum_{(i \rightarrow j, tc) \in Spill} Qspill_{(i \rightarrow j, tc), h} + \\ & \sum_{(j \rightarrow i, tc) \in Spill} Qspill_{(j \rightarrow i, tc), h-tc} + TNF_{i,h} \\ \forall i \in & RESERVOIRS, \forall h \in HOURS \end{aligned}$$

The hydraulic balance of all power plants for each hour:

$$\begin{aligned} Qpu_{u,h} = & \sum_{(i \rightarrow u, tc) \in Der} Qder_{(i \rightarrow u, tc), h-tc} \\ Qpu_{u,h} = & \sum_{(u \rightarrow i, tc) \in Cha} Qcha_{(u \rightarrow i, tc), h} \\ \forall u \in & PU, \forall h \in HOURS \end{aligned}$$

The lower and upper bounds of variables:

$$\begin{aligned} Qpu_{u,h} \leq & Qpu_{u,h} \leq \overline{Qpu}_{u,h} \\ \forall u \in & PU, \forall h \in HOURS \end{aligned}$$

$$\begin{aligned} V_{i,h} \leq & V_{i,h} \leq \overline{V}_{i,h} \\ \forall i \in & RESERVOIRS, \forall h \in HOURS \end{aligned}$$

$$\begin{aligned} Qder_{d,h} \leq & Qder_{d,h} \leq \overline{Qder}_{d,h} \\ \forall d \in & Der, \forall h \in HOURS \end{aligned}$$

$$\begin{aligned} Qcha_{c,h} \leq & Qcha_{c,h} \leq \overline{Qcha}_{c,h} \\ \forall c \in & Cha, \forall h \in HOURS \end{aligned}$$

$$\begin{aligned} \underline{Qopen}_{p,h} \leq & Qopen_{p,h} \leq \overline{Qopen}_{p,h} \\ \forall p \in & Pen, \forall h \in HOURS \end{aligned}$$

$$\begin{aligned} \underline{Qspill}_{s,h} \leq & Qspill_{s,h} \leq \overline{Qspill}_{s,h} \\ \forall s \in & Spill, \forall h \in HOURS \end{aligned}$$

with:

$V_{i,h}$  volume of reservoir i at time h

$\underline{V}_{i,h}, \overline{V}_{i,h}$  lower and upper bounds of  $V_{i,h}$

$Qpu_{u,h}$  production flow of unit u at time h

$\underline{Qpu}_{u,h}, \overline{Qpu}_{u,h}$  lower and upper bounds of  $Qpu_{u,h}$

$Qder_{(i \rightarrow u, tc), h}$  power-pipes discharge at time h from reservoir i to unit u, with run-off time tc

$\underline{Qder}_{d,h}, \overline{Qder}_{d,h}$  lower and upper bounds of power-pipes discharge  $Qder_{d,h}$

$Qcha_{(u \rightarrow i, tc), h}$  channel discharge at time h from unit u to reservoir i, with run-off time tc

$\underline{Qcha}_{c,h}, \overline{Qcha}_{c,h}$  lower and upper bounds of  $Qcha_{c,h}$

$Qopen_{(i \rightarrow j, tc), h}$  penstocks discharge at time h from head water i to collecting reservoir j, with run-off time tc

$\underline{Qopen}_{p,h}, \overline{Qopen}_{p,h}$  lower and upper bounds of  $Qopen_{p,h}$

$Qspill_{(i \rightarrow j, tc), h}$  water spill at time h from head water i to tail water j, with run-off time tc

$\underline{Qspill}_{s,h}, \overline{Qspill}_{s,h}$  lower and upper bounds of  $Qspill_{s,h}$

$TNF_{i,h}$  total natural flow of reservoir i at time h

#### Outline OMA's scheduled consistency check

The first goal of OMA is either **to confirm or to recalculate the day before scheduling**, maybe modified during the day of delivery, **in order to respect all the multi-reservoir system constraints**. If the planned production were feasible, OMA's aim would be the minimization of the water spills. Otherwise, on the one hand the tool provides a practicable schedule, minimizing the deviation from the original plan to the recalculated

one, and on the other hand it continues to minimize the water spills.

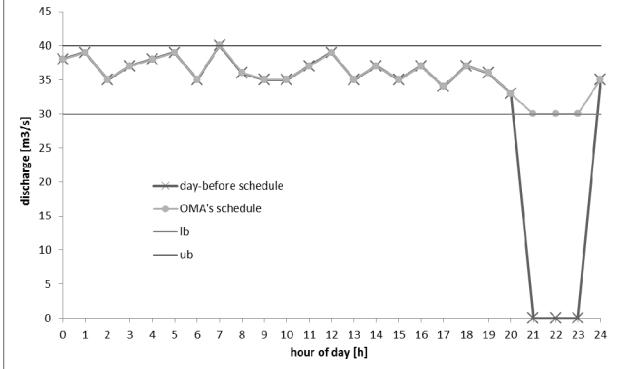


Fig.1 Example of OMA's scheduled consistency check.

In Fig.1 it is compared the day before schedule to the OMA's first step output, rather a new feasible production for a power unit. In this case, the unit is the lower-altitude of the multi-reservoir system, and there is an irrigation constraint in the end of the valley that needs at least 30 m<sup>3</sup>/s of discharge. Near the midnight, the D-1 schedule forces the unit to be off, but the plan is not respecting all the system constraints. Consequently, OMA recalculate a minimum deviation program, keeping on the power plan at the requested water flow. The different final volume of the unit's head reservoir is shown in Fig.2.

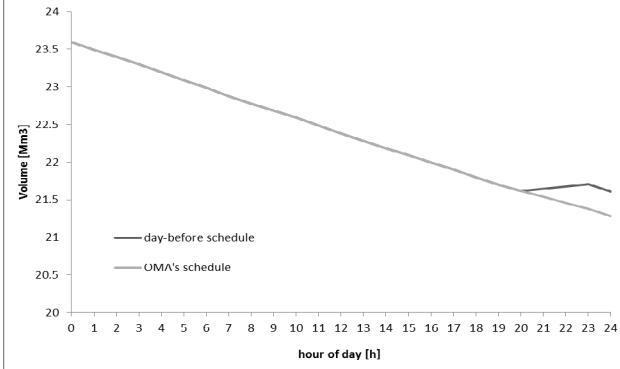


Fig.2 Head water's volume at each time steps.

#### Outline OMA's lower and upper bounds definition

After the scheduled consistency check, confirming or recalculating the production schedule, D+1 started volumes of multi-reservoir system have already been evaluating, as well as every network's discharge.

With the above-mentioned information and the reservoirs' target volumes for all optimized day ahead, it is possible for OMA to compute feasible and operative limits for all the power units in the days after D.

OMA's goal is reachable by several optimizations in cascade, one independent from the others, where the objective function changes. In each one of these, the

object is the water discharge of the power plan  $\bar{u}$  at time  $\bar{h}$ . Minimizing (maximizing) the objective function, preserving the minimization of spills and "turn loose", from the object, all the other variables (for example water discharge of unit  $\bar{u}$  at time  $h \neq \bar{h}$ ), OMA calculate the lower (upper) bound of the water discharge of the power plan  $\bar{u}$  at time  $\bar{h}$ , respecting all the problem constraints.

#### OMA's output

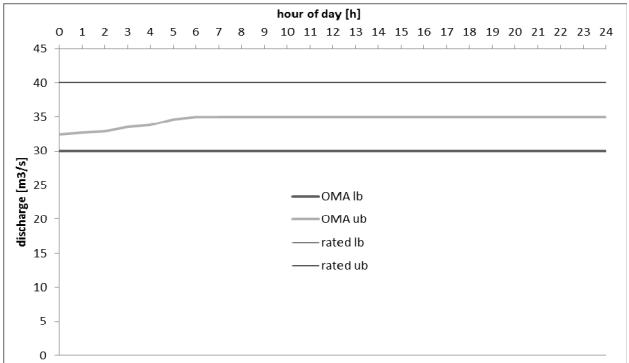


Fig.3 Example of OMA's operative limits definition for a power plant.

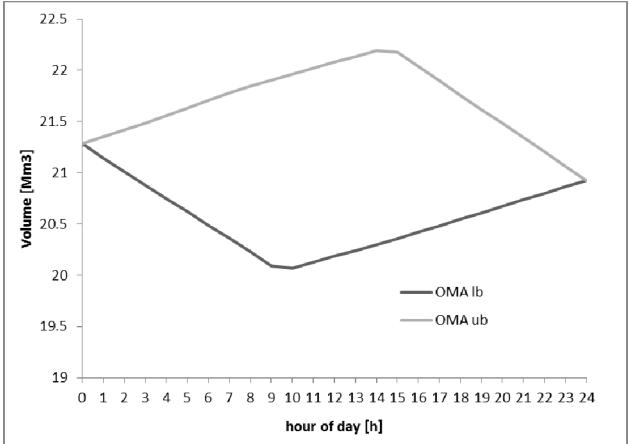


Fig.4 Example of OMA's operative limits definition for a reservoir.

For each power plant and for each time step (generally the time step is not less than 1 hour), **the output of OMA is the definition of the upper and lower bounds** for the production that must be considered during the bidding phase. Fig.3 illustrates the operative limits calculated for the same unit presented in the example of OMA's scheduled consistency check for the day D+1.

Similarly, for each reservoir, the output is the definition of the upper and lower bounds of the water that must be considered stored at the end of each day during the bidding phase, as is shown in Fig.4, where the reservoir is the previously-mentioned head water, with the D+1 start volume equal to the final volume obtained in the first OMA's step.

## Real time operation

### Generalities

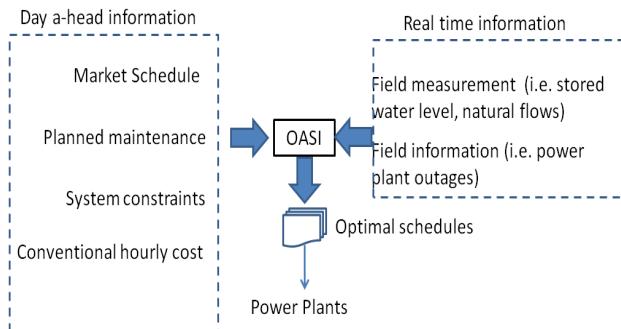


Fig.5 Input and output of OASI [1].

This step of the process has the objective to minimize the economic penalties which are charged according to the market rules, if the power producer is not able to respect the energy plans scheduled by the market.

The Italian electricity market is the set of the Day-Ahead Market (MGP), the Intra-Day Market (MI) and the Ancillary Services Market (MSD) [4]:

- in the MGP phase the electricity is traded through supply offers and demand bids.
- in the MI phase the variations of the volumes of electricity traded in the MGP are negotiated.
- in the MSD, the Italian System Operator Terna procures the resources needed for its dispatching services (i.e. for managing, operating, monitoring and controlling the system), relieving intra-zonal congestions, creating energy reserves and real-time balancing.

Every plant, depending on its characteristics, can participate or less at a market phase. We define a MGP/ MI/ MSD plant, if it participates into MGP/ MI/ MSD phase of the market, as defined in [2].

The process is performed the day of delivery at each regional center.

OASI is the tool used to manage real time operation, interfacing to the real-time measurement system installed on the field.

OASI is designed to work:

- on a routine basis: OASI performs a first optimization of the multi-reservoir system at the arrival of the first market schedule and then a new optimization is repeated every 30 minutes,

in order to continuously track scheduled production with the new real time information.

- on an external event trigger: when there is an external event that needs a new computation - for instance, the sudden outage of a generator or the arrival of new constraints - a new optimization is performed.

Fig.5 shows the input and the output of the application OASI [1].

For each multi-reservoir system, some inputs are collected at the D-1 day or at the hours before the beginning of the optimization. The inputs are market schedule resulting from the bidding strategy, planned maintenance that could make some generators unavailable and system constraints, such as the minimum and maximum level of the water stored in each reservoir. The conventional hourly costs are a representation of the best available forecast of the penalty costs that must be paid if the energy market schedules are not respected. These costs are compared with other penalty factors referred for example to spillways' constraints or basins' volume at the end of the day. Other inputs are real time information, such as the sudden outage of some generators, the real time measurements of the water levels in the reservoirs, and the real time natural flows.

The output of OASI is the optimal schedule of the power plants belonging to the same hydraulic system: **the objective of the optimization is to minimize the economic penalties that are charged, according to the market rules, if the power producer is not able to respect the energy plans scheduled by the market every 15 minutes** of the day of delivery. The output covers the time horizon till midnight with time steps of 15 minutes. After Operator's check, the optimal schedule may be sent to the power plants for real time actuation.

### Problem statement

The objective function to be minimized are penalty costs that must be paid if the hydraulic system is not able to respect the schedule of energy production.

These costs to minimize are composed by the following terms, each ones with a penalizing coefficient (conventional cost):

- the sum of the unbalance from the energy calculated and scheduled of the multi-reservoir system plants.
- a penalization term for the spillways.
- a penalization term for the unbalance respect to the desired final volumes of the basins.

Just the energy schedules for the MSD and MI plants are involved in the minimization process, while the powers

of the others plants (MGP) are free variables for the system.

For each power plant, the electrical power is expressed as a linear function of the outgoing water flow, where the energy factor of power plant is kept constant during the optimization.

The system variables are:

- the discharge of the channels (ingoing and outgoing in/from the reservoirs and in/from the plants)
- the levels of the reservoirs

The description of the hydraulic system is achieved through a system connectivity matrix mapping flow routing within the system.

The hydraulic system constrains are:

- the hydraulic balance of all the reservoirs for each time step
- the maximum daily variation limits of the stored water
- upper and lower bounds of variables
- time derivative upper bound of unit production
- the possible irrigation requests

Like OMA, OASI's problem is linear, and the minimization is solved through an interior point method, which has proved to be reliable for problems with this kind of complexity[3].

#### *A multi-reservoir system as a whole power plants*

Several multi-reservoir systems always have unbalance of the energy scheduled because of either the limited capacity of the basins or environmental reasons.

In this case the model is a bit different: the MSD plants of the multi-reservoir system are aggregated (Fig.6) to set up a whole plant with a only one energy scheduled plan. The output is the production of the each plant to be sent to actuation.

Table 2 shows an example of different results of costs when the plants are aggregated or not.

In the first case, the MSD-plants 2 and 3 are taken individually, while, in the second case, units 2 and 3 are aggregated to set up a plant (plant 1 bids only on MGP market, so it is not involved in the objective function).

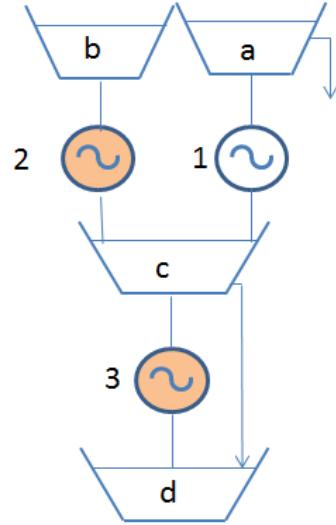


Fig.6 A part of multi-reservoir system: the letters are basins, the numbers are plants, the line are channels and the arrows are spillways.

The hypotheses on the penalty cost factors for the optimization are:

- if there is overproduction, value cost is 40 p.u./MWh
- if there is underproduction, value cost is 30 p.u./MWh
- penalty cost value for the final volume basins (c and d in Fig.2) is 100 p.u./MWh

The above-mentioned hypotheses mean that it is better unbalancing than having not the desired volume.

Table 2 An example of costs result after optimization with aggregated plants or not. The costs are expressed in p.u.

	Single plants	Aggregated plants
Unbalance cost	60	0.6
Volume cost	0.1	0
Total cost	60.1	60

The illustrated example is a real case of multi-reservoir system that needs the model of aggregated plants. This option gives the possibility to respect the hydraulic constraints and to allocate the energy for each plants without unbalancing the aggregated scheduled plan.

## Real time actuation

### *Generalities*

This step of the process is performed the day of delivery and the focus is not the whole multi-reservoir system, but the single hydro power plant, modeled in deeper technical detail. The tool used is OPG, which may be installed

centrally, such as in the regional control center, or locally in each power plant to be controlled.

The main input of OPG is the scheduled 15 minutes energy profile that the power plant must respect for the next 6 hours. This plan might be different from the market schedule, because of the re-scheduling by OASI. Other inputs are field measurements about the real-time power production and the stored water in the reservoirs directly connected to the power plant.

The aim of OPG (Fig.7) is to find the optimal way to manage the power plant by achieving the **minimization of the water consumption, in respect of the provided 15 minutes energy schedule**. Mainly, this means to define the best scheduling of the units, to define the correct time distribution of the generation in the 15 minutes in order to maximize the units' efficiencies, and to define the best management strategy for the regulated channels' sluicegates in order to optimize the basins levels.

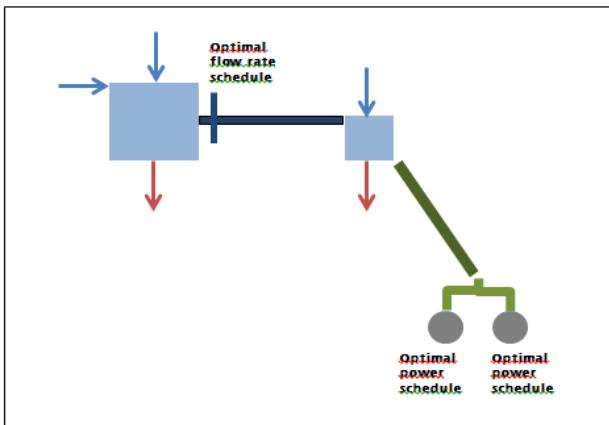


Fig.7 Representation of OPG's goal.

**OPG outputs are the 1-minute-reference values of the electrical power of each unit and the water flow reference of each regulated channel during the next 6 hours.** Since it has already been checked by OASI, the 15 minutes energy schedule is considered by OPG as a very tight constraint.

The output of OPG is directly used for **automatic real time actuation**.

#### Structure of OPG

The OPG tool is based, as Fig.8 illustrates, on a database in which almost all data acquired, all the results produced and all the data structure of the hydraulic systems are temporary or permanently stored. Main inputs are the 15 minutes energy schedules (PV/PVM), the field information acquired by files, and the system data

structure. Preliminary estimation and optimization processes exchange data with the database to provide the optimal electrical power and water flows to the actuators on field.

Optimization process is also fed by field measurement archives.

The optimization activity (preliminary estimation and optimization processes) may be activated by:

- manual request through MMI
- arrival of new PV/PVM
- arrival of new files (unavailability or temporary constraints)
- external conditions, that can be very different from forecasted ones (e.g. wrong natural flow forecast)

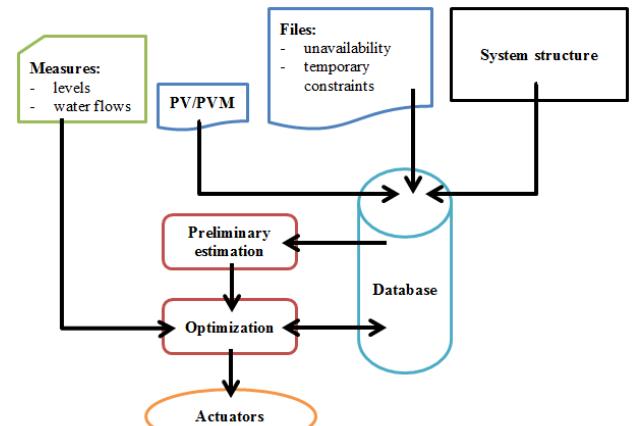


Fig.8 OPG's structure.

#### Hypotheses

The mathematical model of the hydraulic system has been made with the following assumptions:

- startup and shutdown water consumption neglected (considered in future)
- hydraulic efficiency (intended as the ratio between electrical net power and the product of net head and water flow) considered as quadratic function of the water flow and different for each group
- constant discharge level
- quadratic dependency of penstock losses from water flow
- small level variation of reservoirs (linear level/volume curve based on initial condition)
- constant value for open channel time delay
- maximum open channel flow rate proportional to the upstream head

## Preliminary estimation

The optimization method adopted is the **internal point method**, due to its ability to obtain calculation convergence with a reduced iteration number [3] also for medium/big size problems (more than 10000 variables). The optimization algorithm has been implemented in Java language.

This method is also characterized by some limitations that don't allow the resolution of the OPG problem in its wholeness. In fact, with the internal-point method it is not possible to solve non convex problems and mixed-integer problems, but the hydraulic system problem to solve has both these features: **non-convexity**, introduced by hydraulic efficiency equations, and **non-integer variables**, introduced by non-zero minimal power of the single units.

To overcome this issue, a **preliminary estimation** of the solution has been implemented in way to restrict the problem's boundary in a convex region with only continuous variables.

## Optimization

The preliminary estimation process provides a restricted region in which the problem is convex and integer, so proper to be solved in the **optimization** process with the internal-point method.

The **objective function** is aimed to the minimization of the real and equivalent water consumption of  $N_g$  groups in  $t = 1..N$  time step, and it has been defined as follows:

$$\min \left\{ \sum_{t=1}^N \left[ \sum_{i=1}^{N_g} qg_{i,t} + \sum_{i=1}^{N_g} (cdp_{i,t} \cdot dp_{i,t}^2) + \sum_{i=1}^{N_g} (cs_i \cdot qs_{i,t}) \right] + \sum_{h=1}^{N_e} (c \psi u \cdot \psi u_h + c \psi d \cdot \psi d_h) \right\}$$

It is composed by the summation of four terms to minimize:

- total of the real water consumption  $qg_{i,t}$  of all the units to achieve the scheduled energy for the power plant
- penalization term, expressed as equivalent water consumption, for minor water oscillations  $dp_{i,t}$  (small water saving related to non-negligible power oscillations)

- total water loss  $\sum qs_{i,t}$  on spillways. This term may be penalized thanks to a multiplying factor  $cs_i$
- penalization term, expressed as equivalent water consumption, for unbalances ( $\psi u_h$  for the positive unbalance,  $\psi d_h$  for the negative one) respect to the scheduled energy (relaxation of the energy constrain).  $N_e$  is the number of energy constrained time steps.

The **equality constraints** represent the physical description of the problem:

- mass balance on basins and nodes
- time delay for open channels
- head loss on penstocks
- hydraulic efficiency equations
- units power calculation
- power gradient calculation
- deviation from the scheduled energy
- theoretical maximum flow equation for open channels

The **inequality constraints** represent upper and lower bounds of each variable.

## Results

In the following figures are shown some examples of the optimization results.

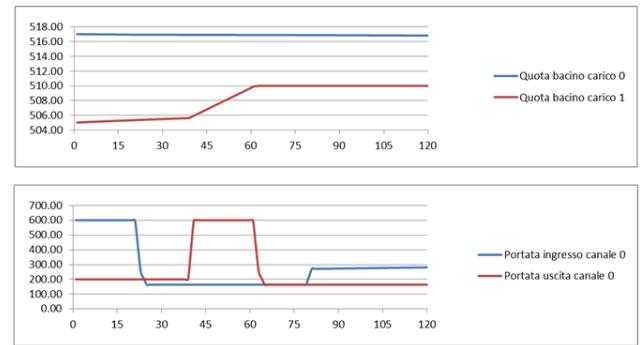


Fig.9 Example of OPG's optimization results(I).

In the second plot of the Fig.9, the blue line represents the income flow of an open channel, controlled by the sluicegate, while the red line the outcome flow resulting from the income flow subject to the time delay of the channel. In the first plot the red line represents the level of the water in the load basin at the end of the channel. It may be noticed that the income flow has been controlled in order to achieve, as soon as possible (after the initial

time delay), the maximum level of the load basin, corresponding to the most efficient condition. After that the water flow has been reduced in order to maintain the maximum level without overflows.

In the Fig.10 are shown the water flow profiles (first plot) and the generated power profiles (second plot) of the two units of a hydro power plant. The two units are similar but have slightly different efficiency curves (third figure). When the load request is smaller (initial and final hours of the simulation) a single unit is enough so only the most efficient unit is working. In the central hours, when the load request is too high to be provided by only one unit, the second one is turned on and the power generation is not shared in equal parts, but distributed in an optimal way, that doesn't necessarily mean greater for the most efficient unit.

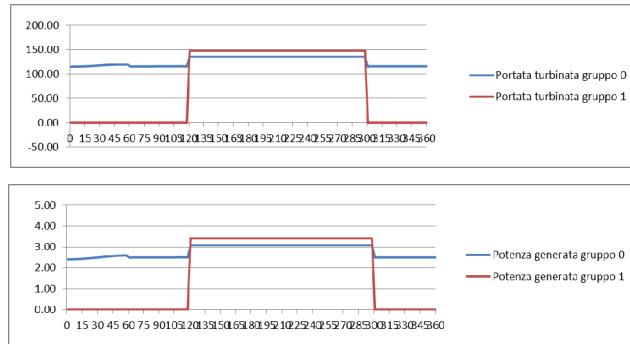


Fig.10 Example of OPG's optimization results(2).

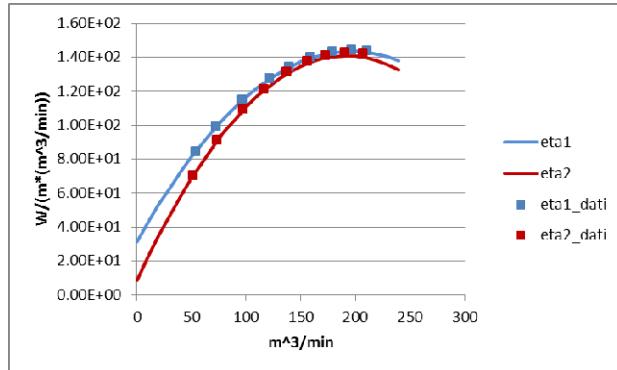


Fig.11 Efficiency curves.

At the moment the OPG tool has been preparing for on-site tests on a regional control center. From the first theoretical estimations, the use of the tool would give especial benefits to the hydro plants in which the regulation of the open channels feeding the loading basin is critical. In these power plants the tool may allow the drastic reduction of the water consumption (up to 10%) caused by not optimal management of the channel sluicegates.

## References

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