2013 IREP Symposium-Bulk Power System Dynamics and Control -IX (IREP), August 25-30, 2013, Rethymnon, Greece

Understanding the Electromechanical Wave Propagation Speed

Urban Rudez University of Ljubljana Faculty of electrical engineering Trzaska 25 1000 Ljubljana Slovenia

Abstract

This paper deals with the phenomenon of electromechanical wave propagation in large power systems. Understanding this phenomenon is necessary for setting up an accurate source localization methodology. At the moment, localization is being performed on the assumption that the electromechanical wave propagation speed is constant - despite the fact that published research has shown that generator inertia constant and transmission line impedance have a strong impact on the speed. Of course, the localization is in general not accurate enough. This is why in this paper the analysis of these influences is presented on a 64-generator ring test power system. Such an analysis may be useful for the formation of sophisticated localization technique, which would consider structure of the studied grid itself.

Introduction

of deals with the phenomenon The paper electromechanical wave propagation in large power systems. Researchers and engineers are able to monitor and study electromechanical waves by taking advantage of Wide Area Measurement Systems (WAMS) and Phasor Measurement Units (PMUs), which are being both widely used in recent years in many power systems around the world. In the past, researchers recognized many potential applications in the field of Wide Area Monitoring, Protection and Control (WAMPAC) [1], which could arise by setting up a solid theory behind electromechanical waves. One of such applications is the determination of a fault location by measuring the electromechanical wave Time of Arrival (TOA) on at least three locations in a power system. By applying the localization techniques (e.g. multilateration) it is said to be possible to determine the location of a fault that caused generators swings.

An exceptional contribution to this topic was made by researchers at the Virginia Polytechnic Institute and State

978-1-4799-0199-9/13/\$31.00 ©2013 IEEE

Rafael Mihalic University of Ljubljana Faculty of electrical engineering Trzaska 25 1000 Ljubljana Slovenia

University by introducing and implementing the so-called FNET, the U.S. national frequency-monitoring network [2]. However, rather than the known and accepted technology of PMUs, which are typically installed in high-voltage busses, they developed much cheaper frequency-disturbance recorders (FDRs), which are, in contrast to PMUs, installed on the 110-V outlets [2]. Nevertheless, these FDRs similarly measure the system frequency from the voltage waveforms and provide the FNET with synchronized, close-to-real-time, frequency information at different locations in a power system.

The results of a fault localization technique, obtained from FNET, are (according to the available literature) based on the assumption that the electromechanical wave propagation speed is constant. Even though the research in a uniform, continuous and isotropic model of a power system has shown that the speed is influenced by several power system parameters [3] and is therefore not constant all over the power system, the constant speed assumption is at the moment the best alternative for the actual situation. So the question remains how to implement the findings regarding variable propagation speed (obtained by setting up a uniform, continuous and isotropic power system model) in a real power system.

In this paper, the most important influential parameters of electromechanical wave propagation speed will be discussed. According to [3] those parameters are mainly generator inertia constants H and transmission line reactances X. In order to represent the impact of variables H and X in the power system as clearly as possible, a 64-generator ring test system is used, which is in its structure a radial system. Even though the actual power systems are highly meshed, performing simulations on such a test system represents a necessary step in the process of understanding the properties of an electromechanical wave. In this paper, simulations were carried out by using the professional power system dynamics tool.

The explanations and results, presented in the paper, will show that measuring the electromechanical wave TOA at a certain point in a power system (discussed in [4]) is not a trivial task. Namely, both H and X have a strong impact on the amplitude as well as the wavelength of a wave and consequently introduce many problems to the process of measuring TOA.

The paper is organized as follows. First a phenomenon of an electromechanical wave travelling will be presented on a 64-generator ring test system. Next, the impacts of generator inertias and transmission lines impedances on the wave propagation speed will be analyzed. According to this analysis the problems with TOA measuring are discussed. Finally, the conclusions are drawn.

Electromechanical wave - phenomenon description

Test system

In this section, the electromechanical wave in a ring power system is discussed in such a way that the reader will gain an insight to the main concept of the phenomenon. Authors are aware that real transmission systems are highly meshed. However, for the reason of simplicity the explanation will base on simulations of the so-called 64-generator ring system, taken from [5] (Figure 1). It comprises 64 serially connected busses, forming a ring (two neighboring busses are connected with a transmission line with a reactance X, as well as the first and the last bus). To each bus, a single synchronous generator is connected. The parameters of all the elements, included in this system, are taken from [5].



Figure 1: 64-generator ring system

The initial phase angle values of the neighboring busses prior to a disturbance are $360^{\circ}/64 = 5.625^{\circ} = \Delta \delta_{init}$ apart. In order to perform the representative simulations, the ring system was slightly modified. Instead of the first synchronous generator, being connected to bus 1, an infinite power source (considered as a phase angle reference) was connected to this bus via an ideal phase shifting transformer (PST). By applying a sudden phase shift $\Delta \delta_{\text{dist}}$ on the bus 1 with respect to the infinite power source phase angle for the time period of $T_{\text{dist}} = 50$ ms, we have simulated a disturbance propagating through the ring in a form of an electromechanical wave (Figure 2).



Figure 2: Generator angles of all 64 generators in a 64generator ring test system

It is obvious according to Figure 2 that the wave travels in both directions from bus 1 (phase angle on bus 1 is drawn with a dotted line). When both waves meet at the far end of the ring, a so-called dispersion effect can be recognized (this can also be referred to as power system oscillations). This can be avoided by implementing a zero-reflection controller ([6]) at any generator bus, which acts as an ideal buffer (the wave does neither continue its path along the line neither does reflect back in the opposite direction). However, in our study we were interested only in the first few moments when the wave reaches a certain bus and therefore did not have the need for such a controller.

Phenomenon insight

Let us take a look at the three-bus section of a 64generator test system, shown in Figure 3. For the sake of simplicity, the generator is represented with a constant voltage source behind the transient reactance, where the latter was set to a very small value. All generators and transmission line reactances have the same parameters, i.e. inertia constant of *i*-th generator equals $H_i = 2$ sec and line reactance of *i*-th line equals $X_i = 160 \Omega$, which corresponds to the line length of approximately 530 km.



Figure 3: Observed three busses in a 64-generator ring system

Figure 4 depicts the phase angle deviations from their initial values on generator busses 9, 10 and 11. In addition, conditions on bus 10 are shown in Figure 5 more detailed. Solid black curve depicts the phase angle deviation $\Delta \delta_{10}$, dashed black curve the acceleration power P_{a10} that causes G10 to accelerate, dark-grey curve the active power P_{X9} on line X_9 and light-grey curve the active power P_{X10} on line X_{10} .



Figure 4: Bus phase angle deviations



Figure 5: Conditions on bus 10

Let us explain the background of the phenomenon. A sudden phase shift on bus 1 causes the active power on lines X_{64} and X_1 to change. In our case, we observe only that part of a wave, which propagates in the ascending direction of the bus numbering. Namely one should note that the ring system is symmetrical and the wave propagating in the opposite direction is identical. So, the phase angle difference between busses 1 and 2 $\Delta \delta_{1-2}$ suddenly changes (in our case increases), which causes the active power flow on X_1 to increase as well according to:

$$P_{X_{i}} = \frac{U_{i} \cdot U_{j}}{X_{i}} \sin\left(\Delta \delta_{i \cdot j}\right) \tag{1}$$

where indexes are equal i = 1 and j = 2. In general, a sudden phase angle step change $\Delta \delta_{\text{dist}}$ on a certain bus influences the active power flows across the entire power system. According to [7] the change in active power flow, originating from a sudden change on e.g. load bus *k* phase angle, is compensated by changes in active power injections on all *n* generator busses in the system according to the corresponding synchronizing power coefficients:

$$P_{k} = P_{k,0} + \sum_{j=1}^{n} P_{skj} \cdot \delta_{kj\Delta}$$

$$P_{el,i} = P_{el,i,0} + \sum_{\substack{j=1\\j \neq i}}^{n} P_{sij} \cdot \delta_{ij\Delta} + P_{sik} \cdot \delta_{ik\Delta}$$
(2)

In (2), P_k is the active power on the load bus k slightly after the disturbance (phase angle step change), $P_{k,0}$ its initial value prior to the disturbance, P_{sxy} is a synchronizing power coefficient between arbitrary busses x and y, $\delta_{xy\Delta}$ is the phase angle difference between the x-th and the y-th bus due to a disturbance, $P_{el,i}$ is the electrical active power of the synchronous machine connected to bus i and $P_{el,i,0}$ its pre-disturbance value. Please note that denotation n is used for generator busses and denotation k for the load busses. In our case we are considering only one load bus and n generator busses. The synchronizing power coefficient between busses x and y is defined as:

$$P_{\text{sxy}} = V_{\text{x}} \cdot E_{\text{y}} \cdot (B_{\text{xy}} \cdot \cos \delta_{\text{xy0}} - G_{\text{xy}} \cdot \sin \delta_{\text{xy0}})$$
(3)

where V_x is the voltage on *x*-th bus, E_y the internal voltage of a generator voltage source connected to *y*-th bus, B_{xy} and G_{xy} the susceptance and conductance between busses *x* and *y* and finally δ_{xy0} the pre-disturbance phase angle difference between busses *x* and *y*. Both voltages (V_x and E_y) are assumed to remain constant for the whole period of observation).

It is clear from (2) and (3) that $\Delta \delta_{dist}$ changes phase angle *differences* only between those busses, which have a direct electrical connection to a disturbance bus (in our case between busses 1 - 2 and 1 - 64). In their absolute values with regards to the phase angle reference bus, others phase angle busses also change, but $\Delta \delta$ between them remains the same. This means that only two generators, being direct neighbors to a disturbance bus, will experience a sudden step change in electrical active

power drawn from its terminals. This electrical active power change is referred to as a generator acceleration power P_{a} . Synchronous machine speed is defined by the swing equation [7] and influenced by P_{a} :

$$\frac{2H}{\omega_{\rm n}} \cdot \ddot{\delta}_{\rm r} = P_{\rm a} \tag{4}$$

where *H* is a generator inertia constant in seconds, ω_n a generator nominal angle speed in rad/s, δ_r rotor angle in rad and P_a generator acceleration power in p.u. (normalized to generator's apparent rated power). It is clear from (4) that a change in machine's electrical power causes the rotor angle to change. However, the change in electrical power should be treated as an *acceleration* of the generator rotor angle. So despite a sudden appearance of P_a , the change of the rotor angle is gradual. At this point, our thinking should switch from considering discrete changes of phase angles to considering continuous changes.

As the test system parameters are chosen in a manner that all generators react identically to a certain disturbance, let us now shift the window of our observation to the section of a system, shown in Figure 3. Due to an appearance of the acceleration power on G9 (P_{a9}), δ_9 begins to gradually increase from its pre-disturbance value. Consequently the phase angle difference $\Delta \delta_{9-10}$ gradually increases and so does the active power P_{X9} on line X_9 . As P_{X9} increases, G10 experiences rotor acceleration and according to (4) δ_{10} gradually increases. This affects back the electrical circumstances in such a way that $\Delta \delta_{10-11}$ increases on the account of $\Delta \delta_{9-10}$. Consequently, P_{X10} starts to take over the power from P_{X9} . It is important to note that the acceleration power, experienced by G10, equals the difference $P_{a10} = P_{X9}$ - P_{X10} . From the generators point of view this can be seen as if G10 starts to take over the electrical burden from G9 - with some time delay.

Following the described explanation, the phenomenon is transferred all along the ring system. Considering the above explanation, one should keep in mind that generator's inertia constant H determines the machine's mechanical reaction to a certain amount of acceleration power $P_{\rm a}$, whereas transmission line reactance X determines the amount of $P_{\rm a}$ transferred from one machine to the next.

Electromechanical wave - propagation speed influential parameters

Propagation speed

After the introduction of the WAMS technology, which makes it possible to analyze the synchronized data from all over large power systems, researchers noticed that the electromechanical wave propagates through the power system with a speed much less than that of the speed of light. In order to investigate this phenomenon in detail, the author in [8] took the following approach. They decided to create a theoretical model of a power system with spatially distributed parameters, which in real power systems do not occur. This means that not only is the line impedance z distributed along the line length l, but so are the generator inertia constant m and the generator reactance x_{g} . At first sight this seems rather unusual, but by using such an approach it is possible to derive two equations for electromechanical travelling-wave propagation for lossless, one-dimensional propagation, which are very much analogous to electromagnetic wave propagation equations:

$$-\frac{\partial\omega}{\partial z} = \frac{1}{k} \cdot \frac{\partial p}{\partial t}$$
(5)

$$-\frac{\partial p}{\partial z} = m \cdot \frac{\partial \omega}{\partial t} \tag{6}$$

In (5) and (6) $\omega = \omega(z, t)$ and p = p(z, t) represent the distributed generator mechanical frequency and the accelerating power on the distributed generator shaft, respectively, dependent on the line reactance z and the time t. The letter k denotes the distributed transmission capacity for a lossless line.

This approach is very theoretical, but it gives us a splendid insight into the phenomenon. An additional step was taken in [3] where a two-dimensional power system model was considered. One of the positive outcomes of such an approach was the derivation of an explicit expression for the propagation speed of an electromechanical wave v, which is said to be:

$$v = \sqrt{\frac{\omega V^2 \sin \Theta}{2h \cdot |z|}}$$
(7)

In (7), ω is the nominal system frequency, Θ is the line impedance angle in radians, V is the voltage amplitude in p.u., h is the inertia constant in seconds and |z| is the line impedance in p.u. per line length. It is clear from (7) that not only do the electrical parameters of the transmission networks influence the propagation speed, but so does the machine's mechanical parameters. From this conclusion it is clear that the electromechanical wave will propagate through the network according to the size of the generators in different areas and impedances among them. Of course, this is valid as long as the bus voltages and the transmission line R/X ratio remain constant. In the authors' opinion, these assumptions are most likely to be true.

However, measurements in a real power system have shown a very interesting fact that propagation speed of an electromechanical wave in one direction is different than the speed in the other direction [9]. In order to study this phenomenon, the influence of generator inertia constant and transmission line impedance are studied in continuation.

Generator inertia constant

Two different aspects of inertia constant impact on the propagation speed can be considered:

- wave propagation from the generator with lower H towards the generator with higher H (see Figure 6) inertia ascent,
- wave propagation from the generator with higher H towards the generator with lower H (see Figure 9) inertia descent.

In the **inertia ascent case**, the results of two simulations are compared. The first simulation was done with inertia constants of all 64 generators set to H = 2 seconds. The second simulation is different only due to higher inertia constant of the generator G10 set to $H_{10} = 6$ seconds. In Figure 6 phase angle deviations on busses 8 to 11 are depicted, with solid curves for the first and with dashed curves for the second simulation.



Figure 6: Generator inertia constant impact on electromechanical wave – inertia ascent

It can be clearly seen that higher *H* of a certain generator in a series causes different increase in the phase angles, which appears as if the wave is delayed. An important thing to understand is that this effect is not related only to G10, but also the previous generator in a series G9 (δ_8 does not differ much due to a change in H_{10}). Figure 7 and Figure 8 depict generator acceleration power and active power on both lines, connected to the generator busses 9 and 10, respectively. An increase in δ_8 causes P_{X8} to increase as well. An acceleration power P_{a9} appears on a generator G9, which starts to accelerate and raise δ_9 (and consequently P_{X9}). However, due to higher $H_{10} = 6$, appeared P_{X9} is higher than in case when $H_{10} = 2$, as δ_{10} increases slower. As a result, P_{a9} is lower and so is δ_9 lower in amplitude. *This appears as if the wave arrives at bus 9 at slightly different moment (Figure 6), which depends on the TOA measurement method.* Namely, in the middle of the δ_9 wave, TOA appears to be higher, whereas in the peak of the wave TOA appears to be lower.



Figure 7: Power conditions on bus 9 - inertia ascent



Figure 8: Power conditions on bus 10 - inertia ascent

Higher P_{X9} increases the acceleration of G10 P_{a10} , which due to a higher H_{10} accelerates slower. This means that δ_{10} increases slower and consequently also P_{X10} . This is an additional factor which increases P_{a10} . However, one should keep in mind that the impact of higher inertia is in this case more influential compared to higher acceleration power. Our simulations were set in such a way that inertia H_{10} was increased by a factor 3, whereas acceleration factor increases by a factor 2 (at its peak value – see Figure 8). This appears as if the wave arrives at bus 10 delayed (Figure 6). Delay time depends on the TOA *measurement method.* Namely, in the middle of the δ_{10} wave, TOA increase appears to be lower compared to TOA increase at the peak of the wave.

In the **inertia descent case**, same two types of results are compared. The only difference is that the first simulation was done with inertia constants of all 64 generators set higher amount of H = 6 seconds. The second simulation differs in setting inertia constant of the generator G10 to $H_{10} = 2$ seconds. Again, in Figure 9 phase angle deviations on busses 8 to 11 are depicted, with solid curves for the first and with dashed curves for the second simulation.



Figure 9: Generator inertia constant impact on electromechanical wave – inertia descent

It can be clearly seen that lower H of a certain generator in a series causes different increase in the phase angles, which appears as if the wave is faster. Again, an important thing to understand is that this effect is not related only to G10, but also the previous generator in a series G9 (δ_8 does not differ much due to a change in H_{10}). Figure 10 and Figure 11 depict the same variables as Figure 7 and Figure 8. An increase in δ_8 causes P_{X8} to increase as well. An acceleration power P_{a9} appears on a generator G9, which starts to accelerate and raise δ_9 (and consequently P_{X9}). However, due to lower $H_{10} = 2$, appeared P_{X9} is lower than in case when $H_{10} = 6$, as δ_{10} increases faster. As a result, P_{a9} is higher and so is δ_9 higher in amplitude. This appears as if the wave arrives at bus 9 at slightly different moment (Figure 9), which depends on the TOA measurement method. Namely, in the middle of the δ_9 wave, TOA appears to be lower, whereas in the peak of the wave TOA appears to be higher.

Lower P_{X9} decreases the acceleration of G10 P_{a10} , which despite this due to a lower H_{10} accelerates faster. This means that δ_{10} increases faster and consequently also does P_{X10} . This is an additional factor which decreases P_{a10} . However, one should keep in mind that the impact of lower inertia is in this case more influential compared to lower acceleration power. Our simulations were set in such a way that inertia H_{10} was decreased by a factor 3, whereas acceleration factor decreases by a factor 2 (at its peak value – see Figure 11). *This appears as if the wave* arrives at bus 10 faster (Figure 9). Arrival time depends on the TOA measurement method. Namely, in the middle of the δ_{10} wave, TOA decrease appears to be lower compared to TOA decrease at the peak of the wave.



Figure 10: Power conditions on bus 9 - inertia descent



Figure 11: Power conditions on bus 10 - inertia descent

Transmission line reactance

Similar than in case of inertia constant, two different aspects of transmission line reactance impact on the propagation speed can be considered:

- wave propagation from the line with lower X towards the line with higher X (see Figure 12) – reactance ascent,
- wave propagation from the line with higher X towards the line with lower X (see Figure 13) – reactance descent.

The explanation of the impact of line reactance variation requires less detailed analysis. In the **reactance ascent case**, the results of two simulations are compared. The first simulation was done with all 64 line reactances set to $X = 120 \Omega$. The second simulation is different only due to higher reactance $X_9 = 160 \Omega$. In Figure 12 phase angle deviations on busses 8 to 11 are depicted, with solid curves for the first and with dashed curves for the second simulation.

Clearly, higher X of a certain transmission line in a series causes different increase in the phase angles, which appears as if the wave propagates slower. However, it is important to understand that this effect is not related only to the generator following the line with higher X, but also the generator, preceding the same line. As δ_8 increases, so does P_{X8} , which causes acceleration power on G9. However, as X_9 is higher the acceleration of δ_9 determines lower P_{X9} . This does not only increase P_{a9} , but also decreases P_{a10} as well. As a consequence, this appears as if the wave arrives at bus 9 sooner and at bus 10 later (Figure 12). Again, the exact TOA depends on the used TOA measurement method.



Figure 12: Transmission line reactance impact on electromechanical wave – reactance ascend

Similarly, the **reactance descent case** can be explained. The first simulation was done with all 64 line reactances set to $X = 160 \Omega$. The second simulation is different only due to lower reactance $X_9 = 120 \Omega$. Figure 13 shows the same variables than Figure 12.

Lower X of a certain transmission line appears as if the wave propagates faster. As δ_8 increases, so does P_{X8} , which causes acceleration power on G9. However, as X_9 is lower the acceleration of δ_9 determines higher P_{X9} . This does not only decrease P_{a9} , but also increases P_{a10} as well. As a consequence, this appears as if the wave arrives at bus 9 later and at bus 10 sooner (Figure 13). Again, the exact TOA depends on the used TOA measurement method.



Figure 13: Transmission line reactance impact on electromechanical wave – reactance descend

TOA measuring

According to the above analysis, the speed of the electromechanical wave is far from being constant in a power system. At the first sight, it seems that this is mainly due to the effect of different inertias of synchronous machines. Namely, one might conclude that a transmission path reactance is directly dependent on the line length. This means that the transmission reactance is a consequence of the line actually being shorter. Consequently, the electromechanical wave propagation speed in geographical length units per time interval should not be influenced much by a line reactance. However, this philosophy is not valid in case of having parallel lines. So the influence of line reactance should not be ignored.

In order to confirm the analysis, presented in the paper, let us examine the three graphs in Figure 14. The upper graph depicts the measured TOA at generators 3 to 28 in a studied ring system. With the thin line the measurements are shown for all inertia constants set to H = 2 seconds and all line reactances set to $X = 160 \Omega$. On the other hand, thick line represents circumstances with higher $H_8 H_{13} = 6$ seconds (second graph) and lower $X_{18} - X_{23} = 80$ Ω (third graph). At this point it is important to note that TOA was obtained by using a constant threshold method [10] with the threshold set to 5°. One should understand that by changing the threshold value, the measured TOA would also change (this issue was already discussed in [4]).

It is evident from Figure 14 that the steepness of the TOA curve increases when the wave propagates through the area with high-inertia generators. On the other hand, the steepness decreases when the wave propagates through the low-impedance area. Nevertheless, the TOA curve steepness remains the same in all areas with the same generator inertias and line impedances.



Figure 14: Visualization of variable propagation speed of an electromechanical wave

Conclusions

With the help of a WAMS system, the phenomenon of an electromechanical wave in a power system can be observed and analyzed. Applications that perform this task are already in use for several years in many power systems. However, the process of localization of the wave source disturbance needs to be upgraded in such a way that the information about the grid under question should be considered. The experiences have shown that by considering the propagation speed as constant, the localization might show itself as relatively unreliable.

The research presented in this paper shown the background of generator inertia constant and transmission line impedance (reactance) impact on the electromechanical wave propagation speed. In order to fully understand this phenomenon, such an explanation may be useful for the formation of sophisticated localization technique, which would consider structure of itself. The theory the studied grid behind electromechanical wave, based on the uniform, continuous and isotropic power system model does give us a very important and valuable knowledge, but does not change the fact that the generators, connected to a power system, have spatially concentrated and not distributed inertia constants. As a consequence, all potential WAMPAC applications should consider the generators as such as well.

References

 Terzija, V., Valverde, G., Deyu Cai, Regulski, P., Madani, V., Fitch, J., Skok, S., Begovic, M.M., Phadke, A., "Wide-Area Monitoring, Protection, and Control of Future Electric Power Networks," Proceedings of the IEEE, vol.99, no.1, pp.80,93, Jan. 2011.

- [2] B. Qiu, L. Chen, V. Centeno, X. Dong, Y. Liu, "Internet based frequency monitoring network (FNET)," Proc. Power Eng. Soc. Winter Meeting, 2001, pp. 1166-71.
- [3] James S. Thorp, Charles E. Seyler, Arun G. Phadke, "Electromechanical Wave Propagation in Large Electric Power Systems," IEEE Trans. On Circuits and Systems – I: Fundamental Theory and Applications, vol. 45, No. 6, June 1998.
- [4] U. Rudez, R. Mihalic, " A Method of Detecting the Time of Arrival for an Electromechanical Wave in Large Power Systems," *paper will be presented in Powertech 2013 conference.*
- [5] Liling Huang, "Electromechanical Wave Propagation in Large Electric Power Systems," Ph.D. dissertation, Virginia Polytechnic Institute and State University, Blacksburg, Virginia, U.S.A., September 2003.
- [6] Lesieutre, B.C., Scholtz, E., Verghese, G.C., "A zero-reflection controller for electromechanical disturbances in power networks", Power System Computation Conference PSCC, Seville, Spain, June2002.
- [7] P. M. Anderson, A. A. Fouad. "Power System Control and Stability." The Iowa State University Press, First Edition, 2007.
- [8] Adam Semlyen, "Analysis of disturbance propagation in power systems based on a homogeneous dynamic model," Power Apparatus and Systems, IEEE Transactions on, vol.PAS-93, no.2, pp.676,684, March 1974.
- [9] K.S. Kook, Y. Liu, M.J. Bang, "Global behavior of power system frequency in Korean power system for the application of frequency monitoring network", IET Gener. Transm. Distrib., 2008, Vol. 2, No. 5, pp. 764-774.
- [10] Andrew J. Arana, "Analysis of Electromechanical Phenomena in the Power-Angle Domain," Ph.D. dissertation, Virginia Polytechnic Institute and State University, Blacksburg, Virginia, U.S.A., December 2009.
- [11] Robert Matthew Gardner, Zhian Zhong, Yilu Liu, "Location determination of power system disturbances based on frequency responses of the system," U.S. Patent 7 765 034 B2, Jul. 27, 2010.
- [12] Shu-Jen Steven Tsai, "Study of Global Power System Frequency Behaviour Based on Simulations and FNET Measurements," Ph.D. dissertation, Virginia Polytechnic Institute and State University, Blacksburg, Virginia, U.S.A., July 2005.