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Chance-Constrained Generation Expansion Planning Based on Iterative Risk Allocation

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Abstract

This paper deals with the development of an efficient iterative method to solve the chance-constrained generation expansion planning (GEP) problem. Reliability in an economic manner is the main criterion when addressing GEP. The algorithm proposed here minimizes the cost of achieving the required reliability. Computational results based on the IEEE 30- and 118-bus test systems are presented. The proposed method decreases the cost in comparison to existing methods for the same reliability level.

Introduction

Power systems currently face various sources of uncertainty such as the integration of intermittent renewable energy sources, increased demand participation, and uncertain load growth. This leads to the need for improved stochastic modelling in the control and operation of power systems.

In vertically integrated power systems, the main objective of the utility planners is to provide reliable power to consumers. Commissioning a new power generation unit is both a time- and cost-intensive task [1] and hence it forces planners to make economic and reliable decisions well ahead of time. In GEP one seeks to determine the number and size of new units to be installed so that the future demand is met at minimum cost. Most of the literature either does not consider uncertainty as a criterion or considers only a deterministic approximate formulation of it. The randomness of the uncertainties can be modelled using chance-constrained programming (CCP) [2].

This paper reviews some relevant earlier work,provides an outline of the more common chance-constrained version of the GEP problem, presents the proposed CCP-based model and solution algorithm, and presents and discusses computational results on two IEEE test systems.

Background

The GEP problem is well-known in power systems planning and it has been addressed by many earlier papers, for example [3], [4], [5], [6], [7]. A review of optimization methods for utility planning is given in [8]. A deterministic formulation of GEP as a linear programming problem was used in [3] to minimize multiple objectives: cost, emissions, and fuel price risk. The need to include uncertainty in the formulation motivated additional research including the following works. The GEP problem with uncertain demand is formulated in [4]. In [9] the GEP is modelled using a Markov chain and the resulting problem is solved using stochastic dynamic programming. A two-stage stochastic programming model for generation and transmission expansion planning was discussed in [5] where the problem formulation included a risk factor in the objective function. The solution method was based on the minimum variance approach [10] which is a well-known approach to minimize the risk in an investment project. The GEP problem in a market-based environment was discussed in [11].

The concept of modelling uncertainty using probabilistic constraints was introduced in [2], and methods for finding deterministic equivalents of the probabilistic constraints were discussed in [12]. A CCP model for the unit commitment problems was proposed in [13], and one such model for the transmission expansion problem with wind and load uncertainties was discussed in [14]. The probability density function of the wind-farm generation is determined and the resultant CCP problem is solved using a genetic algorithm. Stochastic unit commitment was solved in [13] by converting the joint chance constraints into an equivalent deterministic form. An application of CCP to the GEP problem and the benefit of stating reliability criteria using a probability measure versus deterministic ones was presented in [7]. A modified algorithm for solving the probability measure in CCP in fewer iterations was discussed and applied to GEP in [6]. An algorithm using CCP and risk allocation was applied to dynamic systems and its effectiveness were discussed in [15] and [16].

In this paper, the GEP problem for vertically integrated power systems is addressed using CCP. The uncertainty in the load growth is modelled using probabilistic power flow equations. The advantage of using CCP is the ability to use joint probabilistic constraints, that guarantee a prescribed probability level of being satisfied at the optimal solution, unlike when expected value functions are used. The resulting problem is a mixed-integer joint-chance-constrained problem. An iterative solution procedure that suitably weights the importance of the critical buses in the system is presented and used to solve

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the deterministic equivalent of the joint-chance-constrained problem.

This work differs from [3] because the load growth uncertainty is considered. It is also different from [14] and [5] in the formulation of the problem and the solution procedure; indeed [14] solved the chance-constrained transmission expansion problem with a genetic algorithm whereas [5] used two-stage stochastic programming instead of CCP to solve the generation and transmission expansion problem. Finally, while [6] and [13] solve the deterministic equivalent of the CCP formulation using an iterative procedure, the main contribution of this work is an improved iterative algorithm that dynamically adjusts the probabilistic constraints on the critical buses contributing to the failure. This dynamic adjustement can lead to lower-cost solutions compared to the earlier approaches.

Problem Formulation with Uncertainty

In GEP the variables of interest are the number and sizes of new generation units to be installed. The generation expansion problem can be mathematically formulated as follows [6]:

$$\min \sum_{i=1}^{nb} w_i C_{bn} + \sum_{i=1}^{nb} C_{pni} p_{ngi} + \sum_{i=1}^{nb} C_{pei} p_{egi} \tag{1a}$$

$$p_{ngi} + p_{egi} - p_{si} = p_{li} \quad i = 1...nb \tag{1b}$$

$$p_{si} = \sum_j -b_{ij}(\delta_i - \delta_j) \quad i = 1...nb \tag{1c}$$

$$p_{egmin} \leq p_{egi} \leq p_{egmax} \quad i = 1...nb \tag{1d}$$

$$w_i p_{ngmin} \leq p_{ngi} \leq w_i p_{ngmax} \quad i = 1...nb \tag{1e}$$

$$w_i = \{0, 1\} \tag{1f}$$

where
C_{bn}, C_{pn} are the investment and production cost of new units
C_{pe} are the production costs of existing units
p_{ng}, p_{eg} are the active power levels of new and existing units
p_{li} is the load connected to bus i
p_{si} is the net power flow in all the lines connected to bus i
δ_i is the voltage angle at bus i
b_{ij} is the susceptance of the line
nb is the number of buses in the system
w_i is the binary decision variable for new generation at bus i.

In the above formulation (1a) represents the total cost incurred by the expansion model, (1b) is the supply-demand constraint, (1c) is a linearized model of the power flow in all the lines connected to bus i, (1d) and (1e) are the minimum and maximum production limits on existing and new units, and w_i is equal to 1 if a new unit is installed at bus i, and 0 otherwise. The resulting optimziation problem is a mixed-integer linear programming (MILP) problem.

The loading levels p_{li} at the buses indicate the estimated average value of the load. As the actual load may be above or below this average value, a deterministic solution that is feasible for the average value may or may not be feasible for other realizations of the load. The CCP approach models this uncertainty using a constraint requiring that feasibility has to hold with a user-defined probability level α. Thus the power flow equations (1b) are replaced by the probabilistic constraint

$$Pr(\bigcap_{i=1}^{nb}(p_{ngi} + p_{egi} - p_{si} \geq p_{li})) \geq \alpha \tag{2}$$

Equation (2), representing the intersection of nb random events, is a joint probabilistic constraint that is difficult to solve as it involves the computation of a multi-dimensional Gaussian integral. It cannot be converted to an equivalent deterministic form directly but [13] and [7] suggested ways to convert these joint chance constraints into an equivalent deterministic form.

Let us consider the following equivalent constraint to (2) that follows from the fact that sum of the probabilities of an event and of its complementary event equals one:

$$Pr\left\{\bigcup_{i=1}^{nb}(p_{ngi} + p_{egi} - p_{si} \geq p_{li})^c\right\} \leq 1 - \alpha. \tag{3}$$

A sufficient condition for equation (3) to hold is

$$Pr\left\{(p_{ngi} + p_{egi} - p_{si} \geq p_{li})^c\right\} \leq \frac{1-\alpha}{nb}, \tag{4}$$

and therefore the probabilistic power flow equations can be written as

$$Pr\left\{p_{ngi} + p_{egi} - p_{si} \geq p_{li}\right\} \geq 1 - \frac{1-\alpha}{nb}. \tag{5}$$

Equation (5) is the approximate joint probabilistic constraint for (2). If the distribution of the random variable is known, equation (5) can be converted to an equivalent deterministic form. Since power system loads can be assumed to follow a normal distribution,[17] the quantile of the right hand side of equation (5) can be expressed in the form:

$$Pr\left\{\frac{p_{ngi} + p_{egi} - p_{si} - \mu_{fi}}{\sigma_{fi}} \geq \frac{p_{li} - \mu_{fi}}{\sigma_{fi}}\right\} \geq 1 - \frac{1-\alpha}{nb} \tag{6}$$

where μ_{fi} and σ_{fi} are respectively the mean and standard deviation of the load distribution, and hence the term $\frac{p_{li}-\mu_{fi}}{\sigma_{fi}}$ is the standard normal load variable with zero mean and unit variance. Thus the deterministic form of the constraint is

$$\frac{p_{ngi} + p_{egi} - p_{si} - \mu_{fi}}{\sigma_{fi}} = Z_\alpha \tag{7}$$

or equivalently

$$p_{ngi} + p_{egi} - p_{si} = \mu_{fi} + \sigma_{fi} Z_\alpha \tag{8}$$

where Z_α is the inverse cumulative distribution of $1 - \frac{1-\alpha}{nb}$ at each bus.

The Uniform Z-Update Algorithm

Given the desired probability level α, the algorithm for solving the GEP problem using the formulation described above is:

1) Compute an optimal set of new generations by solving the MILP with objective function (1a) and constraints (8) and

(1c)-(1f); in other words the constraint (1b) is replaced by the approximate deterministic equivalent (8).

2) Use a Monte Carlo simulation to estimate the probability achieved with the computed set of new generations by solving an optimal power flow (OPF) with the new set of generation units for a large number of randomly generated load scenarios. Here, w_i is not a variable and is fixed with the values from step 1) and there is no investment cost term in the objective function. The number of feasible cases divided by the total number of scenarios gives the estimate of the probability.
3) If the estimated probability is not satisfactory, then the Z value is updated and the process is repeated untill the target probability is achieved.

The update of the Z value implies a change in the load conditions p_{li} and is likely to have a significant impact on the optimal new generation decisions. The Z update method proposed in [13] is based on the interpolation of the univariate and multivariate variables and the false position method [18]. The GEP problem is initially solved twice with values Z_h and Z_l that are chosen to correspond to probabilities higher p_h and lower p_l than the desired probability α respectively. These probabilities are converted to the corresponding univariate space Z-equivalents Z_1 and Z_2 using the probability distribution function of the random variable. Based on these univariate Z and multivariate probability values, Z_α is updated for the next iteration using the formula:

$$Z_\alpha^{j+1} = Z_l + \left(\frac{Z_\alpha - Z_2}{Z_1 - Z_2} (Z_h - Z_l) \right). \tag{9}$$

The iterative algorithm basically tries to shrink the interval $[Z_h, Z_l]$. We refer to it as the uniform Z-update algorithm because the same Z value is used to update the constraint (8) for every bus.

The Proposed Modified Z-Update Algorithm

The main drawback of the uniform update method is that same Z value is used for all the constraints. The algorithm updates the Z values at all the constraints, by shrinking the interval's bounds according to the target probability. Since the constraints contributing to the failure are not identfied as such,the same value of Z is applied to all the buses,regardless of the impact or role of individual buses on system stability.

The following modified algorithm is proposed to address this drawback. Instead of updating all the bus constraints with the same Z value, i.e., instead of changing the loading conditions equally at all the buses, we propose to update the loading conditions based on the contribution of each bus to the failure. This means that more emphasis is given to the critical buses. Such a strategic allocation of the risk can play a significant role in reducing the cost of expansion.

The outline of the proposed algorithm is similar to that of the uniform Z-update algorithm. The main difference is the identification of the critical buses and the updating of the Z value independently for each bus. Given the desired probability level α, the steps of the algorithm are:

1) Solve the expansion problem twice, once with each of two values of Z_h and Z_l that are respectively higher and lower than Z_α.
2) The probability achieved with each set of new generations computed in step 1) is determined by solving an OPF for various scenarios of load generated using Monte Carlo simulation. The OPF formulation consists of equations (1a) to (1f) but with the values w_i fixed according to the solutions computed in step 1); hence the investment term in (1a) and the constraint (1f) are removed. Let the probabilities of success be P_h and P_l for Z_h and Z_l respectively.
3) Infeasible cases of the OPF imply that there is not enough generation at one or more buses to meet the load, or not enough transfer capacity in the transmission lines. These buses are the critical buses of the system. To identify them, a positive slack variable for each bus and a corresponding penalty are introduced in the constraints and the objective function respectively:

$$\min \sum_{i=1}^{nb} C_{pni} p_{ngi} + \sum_{i=1}^{nb} C_{pei} p_{egi} + \sum_{i=1}^{nb} C_{pen} s_i \tag{10}$$

$$p_{ngi} + p_{egi} - p_{si} = p_{li} - s_i \quad i = 1...nb \tag{11}$$

$$s_i \geq 0 \quad i = 1...nb \tag{12}$$

where C_{pen} is the penalty for the slack variables in the objective function. Equations (10)-(12) are the modified objective function and constraints that replace (1a)-(1b) for the infeasible cases of the OPF.

The slack variables play the role of reducing the load at particular buses so as to make the OPF feasible. Thus these variables identify the critical buses in the system as they are positive only for those buses responsible for failure. The value of C_{pen} is chosen such that the slack variables are positive only if there is no other alternative to make the OPF feasible. Thus the magnitude size of C_{pen} must be relatively high compared to the other costs in the objective function.

4) The multivariate probability values P_h and P_l are converted to their univaritate equivalents Z_1 and Z_2 respectively.
5) Using these values, Z_α^i for bus i is updated using

$$Z_\alpha^i = Z_l + \left(\frac{Z_\alpha - Z_2}{Z_1 - Z_2} (Z_h - Z_l) \right) * \left(\frac{\bar{s}_i}{\hat{s}} \right) \tag{13}$$

where $\bar{s}_i$ is the sum of the slack variables for bus i over all the scenarios, and $\hat{s} = \max_{i=1,...,nb} \bar{s}_i$.

The slack ratio $\frac{\bar{s}_i}{\hat{s}}$ determines the step size for incrementing the Z value for each bus i. The slack ratio at the non-critical buses is zero so for these buses the Z value remains equal to Z_l. At the critical buses, the Z_α value is incremented depending on the relative size of the failure rate. In particular, for a bus i with maximum failure rate

$\bar{s}_i = \hat{s}$, the slack ratio equals one and the change in Z is the maximum.

6) The GEP problem is solved with the updated Z_α^i, then the OPF is solved to determine the probability P_{feas} of the new solution and the corresponding (Z_{new}) is calculated.
7) If $| P_{feas} - \alpha | \leq \triangle\alpha$ then the algorithm terminates. Here, $\triangle\alpha$ is a small tolerance allowed in the target probability. Whenever $P_{feas} > \alpha \pm \triangle\alpha$, i.e the tolerance is not satisfied, the false position method [18] is used to choose the new lower and higher values of Z:
   a) If $Z_{new} < Z_\alpha^i$ then Z_l and Z_2 are replaced with the new Z_α^i and Z_{new} respectively.
   b) If $Z_{new} > Z_\alpha^i$ then Z_h and Z_1 are replaced with the new Z_α^i and Z_{new} respectively.
8) The process is repeated until the target probability is reached.

Thus in this method, Z is updated in small steps given by a combination of both the interpolation of univariate, multivariate values and the slack ratio for each bus. Thus the interpolation is not only based on the target probability and the interval bounds but also a function of the slack ratio which gives information about the relative importance of each constraint. The algorithm assigns a lower Z value at all the non-critical constraints. Hence the loading conditions are updated in a strategic way and total cost is reduced as demonstrated by the computational results in the next section.

Computational Results

The proposed algorithm is applied to the IEEE 30-bus and IEEE 118-bus systems [19] and simulation results are presented and discussed below. The base case loading levels are the same as given in the test system. The IEEE 30-bus system has 30 buses with 41 branches and 6 generator buses. The existing load in the system is 283.4 MW and total existing generation is 335 MW. Reserve capacity of 5% is added to the load and a load growth of 3% per year is assumed. In addition to this, uncertainty in the load growth as a function of confidence level as given by equation (8) is considered. The uncertainty is modelled as a normal distribution satisfying the three standard deviation criterion $3\mu_{fi} = 0.25\sigma_{fi}$ where μ_{fi} and σ_{fi} are the mean and standard deviation of the normalized load growth. The IEEE 118-bus system has 118 buses with 186 branches and 41 generators. The existing load in the system is 3668 MW and the total existing generation is 5804 MW. A 5% reserve capacity is added to the load and load growth is assumed to be 40 % per year.

The simulation uses the system configuration, transmission line characteristics, generators capacity limits and transmission line flow limits as given in the test system. The installation cost for a new unit is \$26,000/MW [20] and the cost of production for both existing and new generation units are taken as 45\$/MWh [20]. The weight of the slack variable C_{pen} should be relatively higher than both the above costs and is taken as \$400,000 in the simulations. For the Monte Carlo simulation 1000 samples of normally distributed demand are used. There is a small boundary of $\pm 0.5\%$ allowed for the probability (P_{feas}) at the end of the iterative algorithm. The optimization problems are mixed-integer non-linear problems and were solved using the BONMIN [21] solver via GAMS [22]. The solver is suitable for mixed integer non-linear programming problems. The simulations were verified for various confidence levels. The computational results show the effect of confidence level in the GEP and the cost difference between the uniform and modified Z-update methods.

Both test systems had sufficient transmission line capacity for the increased load growth. To show the importance of transmission line limits, the line limits of the 30-bus system were modified as follows. The transfer capacity of the line connecting buses 5 and 7 was reduced to 0.45 from 0.7 and the capacity of the line between buses 6 and 8 was reduced to 0.28 from 0.32. The results for the 30 bus system were computed with these modified line limits.

Figures 1 and 2 show the total cost functions of the GEP problem for the two systems using the two different methods of Z-Update. The cost shown includes the installation cost for the new unit and also the operation costs for the new and existing generation. It can be seen that the total cost increases with increasing confidence level. Through this method, users can know the increase in cost incurred by increasing the reliability and thereby inform the user on the choice of the confidence level required. It is clear from Figures 1 and 2 that the new method of Z-update consistently brings down the total cost.

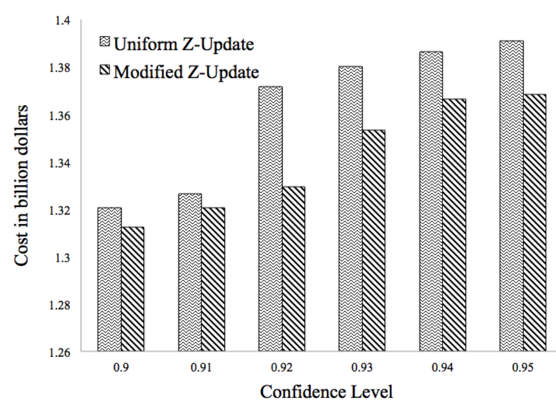

Fig. 1. Comparison of Cost for the IEEE-30 bus test system

Figures 3 and 4 show the variation in Z-values between the two methods for the two test systems. These are the values of Z at the end of the iteration. These figures show how the proposed algorithm focuses on increasing loads only on the critical buses: all the non-critical buses are maintained at Z_l and the critical buses are incremented to achieve the target probability. This difference allows the algorithm to compute lower-cost optimal solutions for the same reliability level.

Tables I and II show the amount of new generation dis-

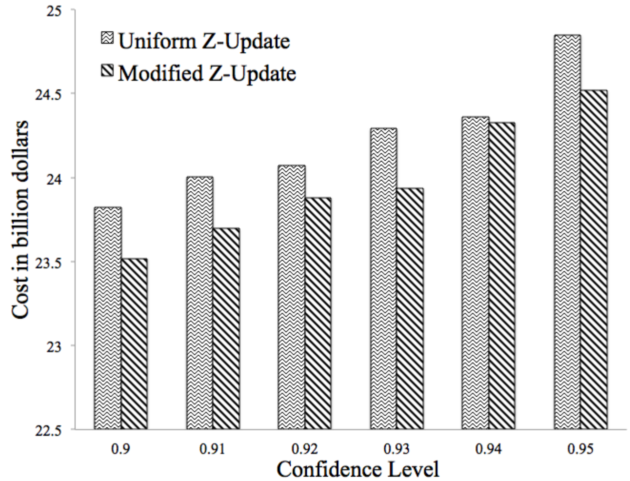

Fig. 2. Comparison of Cost for the IEEE-118 bus test system

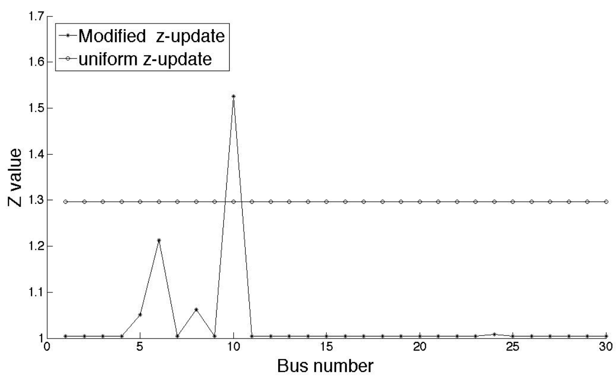

Fig. 3. Comparison of Z-values for the IEEE 30-bus system for 92% target probability

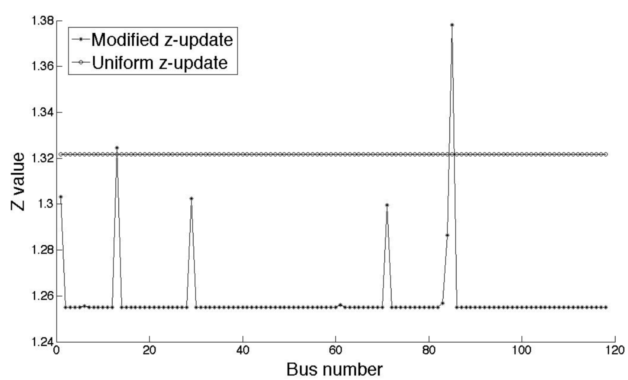

Fig. 4. Comparison of Z-values for the IEEE 118-bus system for 92% target probability

patched for the uncertain load growth corresponding to various confidence levels using the proposed algorithm. Observe that at termination P_{feas} lies within the interval of acceptable probability $\alpha \pm 0.5\%$.

The effect of the modified line limits in the 30-bus system is seen from the dispatch of existing generation in Tables I and II. In Table II, corresponding to the 118-bus system, the existing units are always dispatched to their maximum whereas in the 30-bus system, they are dispatched less because of the modified line limits. The dispatch from existing units keeps increasing up to a confidence level of 93% after which it starts decreasing due to the line overloads.

TABLE I

RESULTS OF GEP FOR THE IEEE 30-BUS TEST SYSTEM

Target probability (α)	Demand	Existing Unit	New Unit	Cost ($*10^6$\$)	P_{feas}
0.90	3.3033	3.2640	0.0393	1.3124	0.8950
0.91	3.3162	3.2644	0.0518	1.3207	0.9070
0.92	3.3299	3.2649	0.0649	1.3295	0.9240
0.93	3.3666	3.2657	0.1008	1.3533	0.9300
0.94	3.3858	3.2643	0.1215	1.3663	0.9420
0.95	3.3860	3.2642	0.1218	1.3664	0.9460

TABLE II

RESULTS OF GEP FOR THE IEEE 118-BUS TEST SYSTEM

Target probability (α)	Demand	Existing Unit	New Unit	Cost ($*10^7$\$)	P_{feas}
0.90	59.0179	58.04	0.9773	2.3519	0.8990
0.91	59.2876	58.04	1.2476	2.3696	0.9080
0.92	59.5704	58.04	1.5313	2.3881	0.9210
0.93	59.6585	58.04	1.6185	2.3938	0.9260
0.94	60.2528	58.04	2.2128	2.4327	0.9450
0.95	60.5466	58.04	2.5066	2.4519	0.9460

Conclusion

A chance-constrained GEP formulation with load uncertainties was considered and a modified algorithm for solving its deterministic equivalent was developed. Slack variables were introduced in the problem formulation to identify critical buses of the system and risk allocations are updated only for these buses. The effectiveness of the new method is demonstrated by computational results using the IEEE 30- and 118-bus test systems. The developed algorithm provides lower total expansion costs than the previous methods.

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