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Association of the Stability Region with Time Scale Analysis to Study Voltage Stability

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Abstract

The problem of voltage stability (VS) due to large perturbations is conceptually very similar to the problem of transient stability (TS). After a large perturbation, the system will be stable if the state of the system, at the time the perturbation is cleared, is inside the stability region (SR) of the post disturbance system. However, the problem of VS has some peculiarities. Models for dynamic analysis of VS have multi-time scale properties and problems of TS and VS may coexist.

VS problems are usually associated with slow dynamics and a QSS (Quasi-Steady State) model is usually employed to study this phenomenon. However, VS problems in the slow variables might be triggered by perturbations in the fast variables and TS problems might coexist with VS ones. In an effort to understand these phenomenon and extend direct methods to the analysis of VS due to large perturbation, we use time-scale decomposition to study problems of VS that are triggered by large perturbations such as short-circuits. In particular we explore a decomposition of the SR and stability boundary of singularly perturbed systems into the SRs and stability boundaries of the slow and fast systems to gain insight into the dynamics of the system and to understand its unstable modes.

Some Basic Concepts

A general model of a power system (PS) for stability analysis is given in the form of the following set of differential algebraic equations (DAE).

$$(\Sigma_\varepsilon)\begin{cases} \dot{\mathbf{z}} = \mathbf{h}(\mathbf{x},\mathbf{z},\mathbf{y}) \\ \varepsilon\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{z},\mathbf{y}) \\ \mathbf{0} = \mathbf{g}(\mathbf{x},\mathbf{z},\mathbf{y}) \end{cases} \qquad (1)$$

In (1), "ε" is a small positive constant and "$\mathbf{x}$" represents variables with fast dynamics, such as angular speed of generators, Automatic Voltage Regulators, etc., "$\mathbf{z}$" represents variables with slow dynamics, such as variations of loads and generation, and "$\mathbf{y}$" is a vector of algebraic variables, including voltage magnitude and angle at network buses.

With $\mathbf{x}\in R^n$, $\mathbf{z}\in R^m$ and $\mathbf{y}\in R^p$, functions $\mathbf{f}$, $\mathbf{h}$ and $\mathbf{g}$ are of class C^1. Let $\varphi_\varepsilon(t,x_0,z_0,y_0)$ denote a trajectory of (1) starting at (x_0,z_0,y_0). For system (1), the algebraic equations $0=g(x,y,z)$ are known as constraint equations and they constraint the dynamics of (1) to the algebraic manifold $L=\{(x,y,z): 0=g\}$.

Let (x_s,z_s,y_s) be an asymptotically stable equilibrium point (ASEP) of (1) and let

$$A_\varepsilon(x_s,z_s,y_s)=\{(x,z,y)\in L : \varphi_\varepsilon(t,x,z)\to(x_s,z_s) \text{ as } t\to\infty\} \qquad (2)$$

be the stability region (SR) of (x_s,z_s,y_s), we define the bound of A_ε by $\partial A_\varepsilon(x_s,z_s)$ [4, 7]. Figure 1 gives a geometrical interpretation of (2).

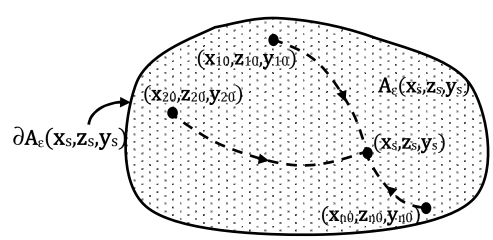

Fig. 1: Stability region and stability boundary of (x_s,z_s,y_s)

The application of singular perturbed systems theory [4, 5, 6] allows the analysis of a PS in the context of time scales (slow and fast).

1. Slow System

The slow system is obtained letting $\varepsilon\to 0$ in (1):

$$(\Sigma_0)\begin{cases} \dot{\mathbf{z}} = \mathbf{h}(\mathbf{x},\mathbf{z},\mathbf{y}) \\ \mathbf{0} = \mathbf{f}(\mathbf{x},\mathbf{z},\mathbf{y}) \\ \mathbf{0} = \mathbf{g}(\mathbf{x},\mathbf{z},\mathbf{y}) \end{cases} \qquad (3)$$

The algebraic equations $\mathbf{0}=\mathbf{f}(\mathbf{x},\mathbf{z},\mathbf{y})$ adds to the constraints of the slow dynamics to the set:

$$\Gamma=\{(x,z,y)\in R^n x R^m x R^p : g(x,z,y)=0;\ f(x,z,y)=0\}\subset L$$

Let $(\mathbf{x}_s,\mathbf{z}_s,\mathbf{y}_s)$ be an ASEP of (3), if $\varphi_0(t,\mathbf{x}_0,\mathbf{z}_0,\mathbf{y}_0)$ is a trajectory of (Σ_0) that starts in the point $(\mathbf{x}_0,\mathbf{z}_0\mathbf{y}_0)$ at t=0, we define:

$$A_0(\mathbf{x}_s,\mathbf{z}_s,\mathbf{y}_s)=\{(\mathbf{x},\mathbf{z},\mathbf{y})\in\Gamma : \varphi_0(t,\mathbf{x},\mathbf{z},\mathbf{y})\to(\mathbf{x}_s,\mathbf{z}_s,\mathbf{y}_s) \text{ as } t\to\infty\} \quad (4)$$

as the stability region of $(\mathbf{x}_s,\mathbf{z}_s,\mathbf{y}_s)$ related to (Σ_0).

2. Fast System

To explore the fast time-scale property of system (Σ_ε), we define the fast time scale $\tau=t/\varepsilon$. In this new time scale, system (1) becomes:

$$(\Pi_\varepsilon)\begin{cases}\dfrac{d\mathbf{z}}{d\tau}=\varepsilon\mathbf{h}(\mathbf{x},\mathbf{z},\mathbf{y})\\ \dfrac{d\mathbf{x}}{d\tau}=\ \mathbf{f}(\mathbf{x},\mathbf{z},\mathbf{y})\\ \ \mathbf{0}=\ \mathbf{g}(\mathbf{x},\mathbf{z},\mathbf{y})\end{cases} \qquad (5)$$

Let $\Phi_\varepsilon(\tau,\mathbf{x}_0,\mathbf{z}_0,\mathbf{y}_0)$ denote the trajectory of (Π_ε) starting in $(\mathbf{x}_0,\mathbf{z}_0,\mathbf{y}_0)$ at t=0. It holds that $\Phi_\varepsilon(\tau,\mathbf{x}_0,\mathbf{z}_0,\mathbf{y}_0)=\varphi_\varepsilon(\varepsilon\tau,\mathbf{x}_0,\mathbf{z}_0,\mathbf{y}_0)$. The fast system (Π_0) is formally obtained forcing $\varepsilon=0$ in (5). In the fast subsystem, variable $\mathbf{z}$ is considered as constant and the fast system (Π_0) can be considered as a family of fast systems (FS) depending on the value of $\mathbf{z}$:

$$\Pi_{FS}(\mathbf{z})\begin{cases}\dfrac{d\mathbf{x}}{d\tau}=\mathbf{f}(\mathbf{x},\mathbf{z},\mathbf{y})\\ \mathbf{0}=\ \mathbf{g}(\mathbf{x},\mathbf{z},\mathbf{y})\end{cases} \qquad (6)$$

Variable $\mathbf{z}$ is considered a constant ($\mathbf{z}=\mathbf{z}^*$) in (6) and is treated as a parameter. If $\Phi_0(\tau,\mathbf{x}_0,\mathbf{z}^*,\mathbf{y})$ is a trajectory of (Π_0) that starts in $(\mathbf{x}_0,\mathbf{z}^*,\mathbf{y})$, we define,

$$A(\mathbf{x}^*,\mathbf{z}^*,\mathbf{y}^*)=\{(\mathbf{x},\mathbf{z}^*,\mathbf{y})\in L : \Phi_0(\tau,\mathbf{x},\mathbf{z}^*,\mathbf{y})\to(\mathbf{x}^*,\mathbf{z}^*,\mathbf{y}^*) \text{ as } t\to\infty\} \ (7)$$

as the stability region of the ASEP $(\mathbf{x}^*,\mathbf{z}^*,\mathbf{y}^*)$ of $\Pi_{FS}(\mathbf{z}^*)$.

The instability of the fast system implies the instability of the complete original system, but the stability of the fast system is not a sufficient condition for the stability of the slow system. Actually, the instability of the slow system might be triggered by the fast dynamics. In the next section, we will illustrate this phenomenon and the relationship between the SR of the slow and fast system with that of the original system using a simple power system model.

A Four Bus System

A four bus system will be employed to illustrate the time-scale decomposition and the decomposition of the SR and stability boundary into the stability region and stability boundary of the fast and slow subsystems.

System of fig. 2 is composed of an infinite bus, a generator at bus 1, which is modeled by the classic model of generators, and a dynamic load at bus 4. The set of algebraic-differential equations of this system is shown in (I), where "x" is a variable associated with the recovery load model [2,3], with T_L=30s and P_0=0.54pu, that are the time constant and nominal load value at t=0s, respectively. The time constant "T_L" of the load is considerably large compared to the other constants then "x" is a slow variable, while "w" and "δ" are fast variables and "V_i" and "θ_i" are algebraic or instantaneous variables. We'll consider that $\varepsilon=1/T_L$ such that (I) is in the form of (5).

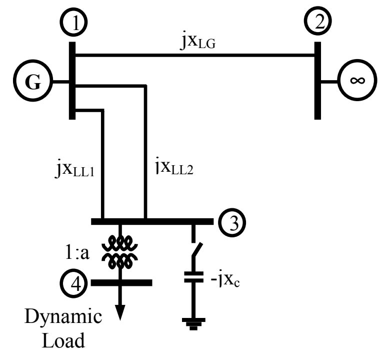

Fig. 2: Four-bus system. The system parameters are: P_m=0.54pu, M=0.0146s, D=0.01, E_g=1.0861pu, x_g=0.5pu, x_{LG}=0.7pu, x_{LL1}=0.8pu, x_{LL2}=0.48pu, a=1.0125, x_c=9pu, E_∞=0.98pu, θ_∞=0, T_L=30s, P_0=0.54pu.

$$(\mathrm{I})\left\{\begin{array}{ll}
\dfrac{dx}{dt}=\dfrac{1}{T_L}(P_0-P_L) \qquad \text{with} \qquad P_L=x+P_0a^2V_3^2 & \text{Dynamic Load}\\
\hline
\dfrac{d\delta}{dt}=w & \\
\dfrac{dw}{dt}=\dfrac{1}{M}\left(P_m-\dfrac{E_gV_1}{x_g}\sin(\delta-\theta_1)-Dw\right) & \text{Generator}\\
\hline
0=V_1\displaystyle\sum_{k=1}^{3}V_k(G_{1k}\cos\theta_{1k}+B_{1k}\sin\theta_{1k})-\dfrac{E_gV_1}{x_g}\sin(\delta-\theta_1) & \\
0=V_1\displaystyle\sum_{k=1}^{3}V_k(G_{1k}\sin\theta_{1k}+B_{1k}\cos\theta_{1k}) & \\
\qquad -\dfrac{E_gV_1}{x_g}\cos(\delta-\theta_1)+\dfrac{V_1^2}{x_g} & \text{Network Equations}\\
0=V_3\displaystyle\sum_{k=1}^{3}V_k(G_{3k}\cos\theta_{3k}+B_{3k}\sin\theta_{3k})+P_L & \\
0=V_3\displaystyle\sum_{k=1}^{3}V_k(G_{3k}\sin\theta_{3k}+B_{3k}\cos\theta_{3k})-\dfrac{V_3^2}{x_c} &
\end{array}\right.$$

A short circuit occurs in the line LL2, very near to bus 1, at t=0s. Protection acts at t=120ms, isolating the shorted line. The fast dynamics is stable but for this clearing time but, after that, a progressive slow decline in voltages due to the influence of load dynamics, which recovers the consumed power, is observed, see fig. 3. At t=90s, a capacitor has to be connected to bus 3 in order to avoid voltage collapse, and stability is restored.

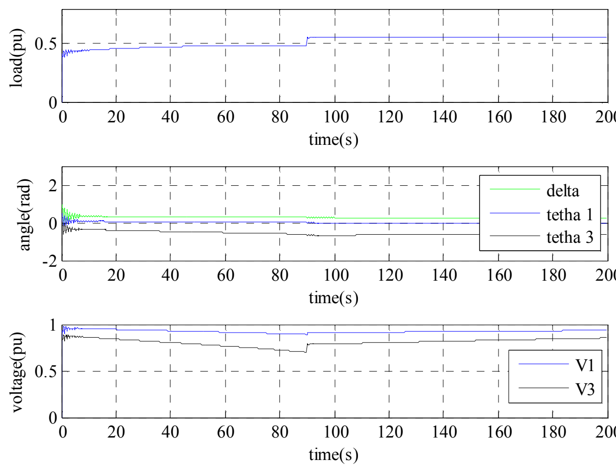

Fig. 3: Step-by-step integration of a contingency of system (I)

System (I), after the shorted line is opened, has no ASEP. Consequently, the concept of SR cannot be used to compute the critical clearing time (CCT). However, if we decompose the system into fast and slow dynamics, then the fast system, defined by ($\Pi_0(x^*)$), after the shorted line is open, considering x*=0, has an ASEP whose stability region is depicted at fig. 4. Actually for every x* there exists one fast subsystem and its corresponding SR and in general the stability of the fast subsystem should be analyzed after a perturbation in the PS. Furthermore, it can be shown that this family of SR of the fast system Cartesian product with the set Γ gives an approximation of the SR of the original system (1) for sufficiently small ε>0 [8].

$$
(\Pi_0(x^*))\left\{
\begin{array}{ll}
\frac{d\delta}{dt} = w & \\
\frac{dw}{dt} = \frac{1}{M}\left(P_m - \frac{E_g V_1}{x_g}\sin(\delta-\theta_1) - Dw\right) & \text{Generator} \\
\hline
0 = V_1 \sum_{k=1}^{3} V_k(G_{1k}\cos\theta_{1k} + B_{1k}\sin\theta_{1k}) & \\
\qquad - \frac{E_g V_1}{x_g}\sin(\delta-\theta_1) & \\
0 = V_1 \sum_{k=1}^{3} V_k(G_{1k}\sin\theta_{1k} + B_{1k}\cos\theta_{1k}) & \\
\qquad - \frac{E_g V_1}{x_g}\cos(\delta-\theta_1) + \frac{V_1^2}{x_g} & \text{Network Equations} \\
0 = V_3 \sum_{k=1}^{3} V_k(G_{3k}\cos\theta_{3k} + B_{3k}\sin\theta_{3k}) + P_L(x^*) & \\
0 = V_3 \sum_{k=1}^{3} V_k(G_{3k}\sin\theta_{3k} + B_{3k}\cos\theta_{3k}) - \frac{V_3^2}{x_c} &
\end{array}
\right.
$$

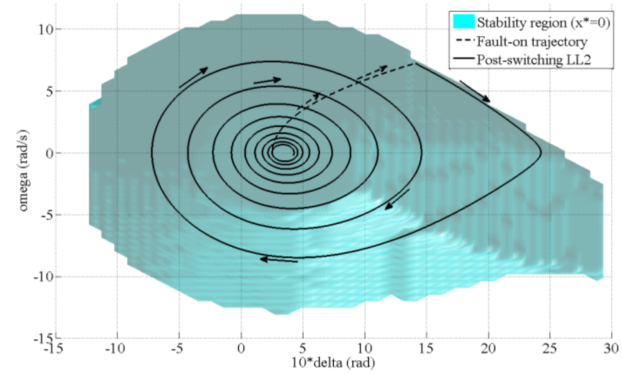

Fig. 4: Phase portrait of the fast dynamics and the stability boundary of the fast system (x*=0).

The slow system is defined by (Σ_0). We claim that the stability of the fast system is not enough to guarantee stability of the slow variables; however the instability of the fast system is a sufficient condition to claim that the complete system is unstable.

$$
(\Sigma_0)\left\{
\begin{array}{ll}
\frac{dx}{dt} = \frac{1}{T_L}(P_0 - P_L) \qquad \text{with} \qquad P_L = x + P_0 a^2 V_3^2 & \text{Dynamic Load} \\
\hline
w = 0 & \\
0 = P_m - \frac{E_g V_1}{x_g}\sin(\delta-\theta_1) & \text{Generator} \\
\hline
0 = V_1 \sum_{k=1}^{3} V_k(G_{1k}\cos\theta_{1k} + B_{1k}\sin\theta_{1k}) - \frac{E_g V_1}{x_g}\sin(\delta-\theta_1 & \\
0 = V_1 \sum_{k=1}^{3} V_k(G_{1k}\sin\theta_{1k} + B_{1k}\cos\theta_{1k}) & \\
\qquad - \frac{E_g V_1}{x_g}\cos(\delta-\theta_1) + \frac{V_1^2}{x_g} & \text{Network Equations} \\
0 = V_3 \sum_{k=1}^{3} V_k(G_{3k}\cos\theta_{3k} + B_{3k}\sin\theta_{3k}) + P_L & \\
0 = V_3 \sum_{k=1}^{3} V_k(G_{3k}\sin\theta_{3k} + B_{3k}\cos\theta_{3k}) - \frac{V_3^2}{x_c} &
\end{array}
\right.
$$

For system (I), fig. 5 shows the locus of the equilibrium points of the fast dynamics and zeros of the algebraic equations (equilibriums of the fast system) for two cases: with and without reactive compensation and the locus of the equilibriums of the slow dynamic (equilibriums of the load), the intersection of those curves gives us the equilibrium points of system (I) or (Σ_0). As we can see, before reactive compensation, the system does not have equilibrium points and after compensation, two equilibrium points appear: A an ASEP and B an unstable equilibrium point (UEP).

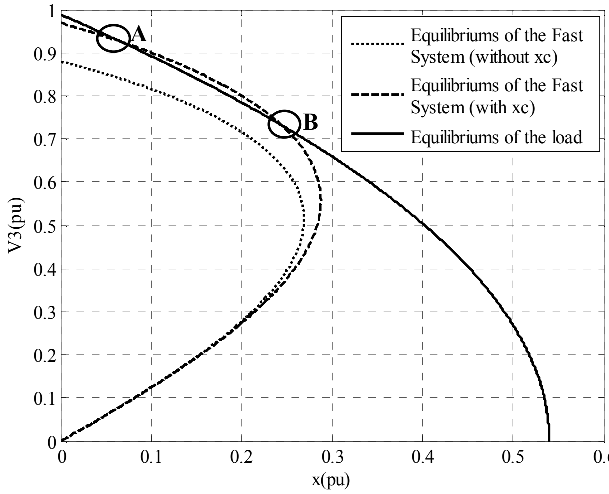

Fig. 5: Equilibriums points of the system

The stability boundary of the ASEP A is shown at fig. 6. QSS analysis and direct methods can be used to estimate the critical switching time (CST) of the capacitor for the slow system, which in this case equals 107s. The stability boundary of the slow system is ε-close to the stability boundary of the complete system [8]. Therefore, if the switching of the capacitor is made at a time smaller than CST, a two-time scale analysis proves that the complete system will be also stable.

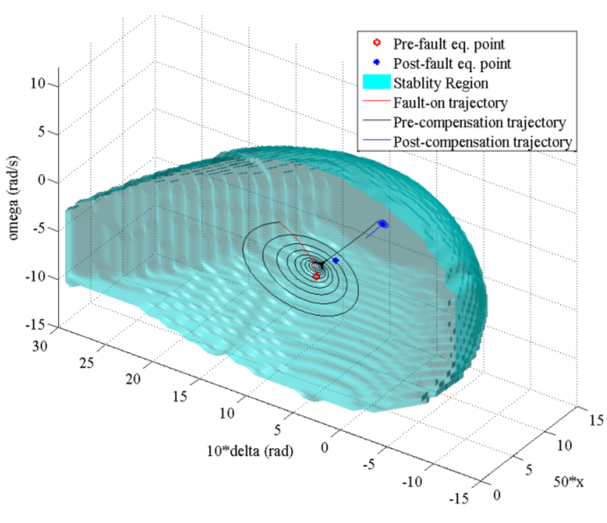

Fig. 6: Stability Boundary of the system after reactive compensation. The axis "x" (slow dynamic) was scaled by 50 in order to improve visualization.

As we can see in (Σ_0), the equations that represent the equilibriums of the fast system are part of the set of constraint equations of the slow dynamic. This fact gives an important relationship between the systems, which is shown in fig. 7. For every x*, we have a fast subsystem (boundary layer system) that has its equilibrium point in the constraint manifold. The stability of this equilibrium point must be verified during the QSS simulation and, especially after a perturbation or actuation of protection or control devices, since after these events the fast dynamics are excited, the stability region has to be determined and stability of the fast transient trajectory has to be checked. In the case of system (I), oscillations in the trajectory (fig. 3) after the action of protection at t=120ms and reactive compensation at t=90s show the excitation of the fast dynamics.

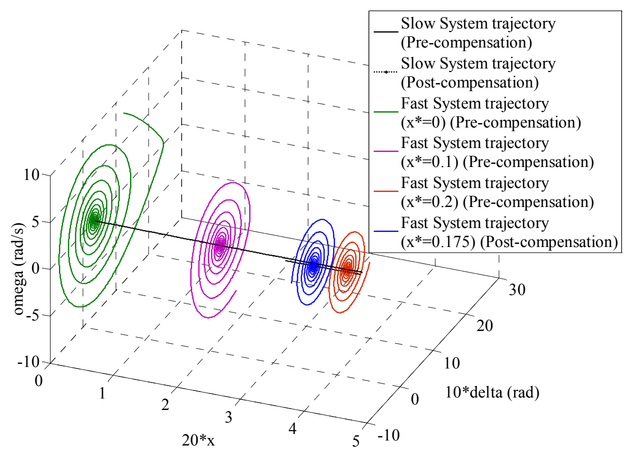

Fig. 7: Relationship between the fast and slow system (x scaled by 20).

The QSS simulation gives in general results with good approximation in relation with the full dynamic system simulation [5]. In the four bus system for example, we obtain that a CST belongs to the interval [103s,107s) when the full dynamic system (I) is used. This "time interval" can be explained because of a relationship between the clearing time of the breaker and the CST. On the other hand, for the QSS simulation, the CCT obtained is 107s. When we make ε=0 to obtain the slow system (Σ_0), we consider the transient phase of the original system stable, then the relationship between the clearing time and CST is lost. In [4, 5], approximations of greater order are suggested for the slow manifold in order to reduce this error, but this also increases the computational effort in the QSS simulation.

Usually, the analysis of stability of the power system in the time domain is broken down in the stability analysis of two-time-scales power system models, like the power system model (I), into transient stability analysis (TSA), for rotor angle stability assessments, and QSS analysis, for the assessment of voltage stability. In the rest of the section, we discuss about the application of direct methods and in particular the CUEP method to compute the CCT of the breakers and CST of the reactive compensation when the CUEP method is applied in the fast and slow system respectively. It's shown the CUEP method can be applied independently for each subsystem.

In [8], the CUEP method for two-time scale (TTS) analysis of PS was proposed. A relationship between the CUEP of the TTS systems and the family of boundary layer systems $\Pi_{FS}(z)$ and the slow system was studied. It was shown a relationship between the CUEP of the original system and the CUEP of the slow and fast systems. Next we apply the two-time-scale CUEP method to study stability of the simple power system (I).

A. Transient stability analysis (fast system)

This kind of analysis is related with the fast system where the slow dynamics are frozen and the fast system is represented by ($\Pi_0(x^*)$). For purposes of analyzing the stability via direct methods, we need to consider the following numeric energy function:

$$V_{FAST} = \frac{Mw^2}{2} - \int_{\delta_0}^{\delta} \left(P_m - \frac{E_g V_1(s)}{x_g} \sin(\delta - \theta_1(s)) \right) ds$$

The CUEP method [1, 7] uses the value of the energy of the controlling unstable equilibrium point to obtain an estimation of the CCT. The critical energy value is V_{fast}=0.4797 and the CCT estimated obtained for this critical value of energy is 189.5ms. A conservative estimation for the CCT of the simplified model ($\Pi_0(x^*)$) was obtained in agreement with the result obtained by the analysis of the original PS (I), where the CCT obtained via numerical integration is 209ms.

Transient stability is not a guarantee of the mid-term stability [5]. Actually, although the simplified system presents an asymptotically stable equilibrium point after the trip of the protection, the original system does not. As we can see in figure 3, the original system, after the trip of the protection, has an unstable mode characterized by a slowly decreasing of the voltage at the load bus due to the recovery load dynamic. As a consequence, stability analysis of the slow system is required.

B. Quasi Steady State Stability (slow system)

QSS analysis is usually performed to the assessment of dynamical voltage stability analysis [5]. In this analysis, it is assumed that the transient phase is 'stable' and the differential equations that model the dynamics of the fast variables are substituted by equilibrium equations (algebraic equations) as we described above.

Although direct methods have been developed for transient stability analysis, they can be conceptually employed to assess the stability of the simplified power system model (Σ_0). For this purpose, one considers the following energy function for the assessment of the QSS model:

$$V_{SLOW} = -\int_{x_0}^{x} (P_0 - P(s)) ds$$

The CUEP of the simplified slow system results z_{os}=(0.2466,0.2831,0.0,0.0,-0.7299,0.89,0.728) and the critical energy of the slow system is V_{slowcr}=0.0014 with a corresponding critical time t_{crs}=106.98s. This time is a good estimation of the CST. Again, there is a good agreement between the results that are obtained between the analysis of the original system and the simplified system (Σ_0), excluding of course the transient in the very first seconds. In particular, numerical integration of the slow system indicates that the CST for the capacitor is 107s.

Conclusions

Stability in QSS analysis cannot guarantee stability of the original system by itself. In spite of applying numerical integration of the original system to study its stability, we can explore the relationship between the CUEP of the original model and the CUEPs of the simplified systems to conclude about stability of the original system. The advantages of doing that are less computational effort to calculate the CUEPs of the simplified systems and avoidance of numerical problems in the calculation of CUEPs.

Acknowledgments

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