Model-free Voltage Stability Assessments via Singular Value Analysis of PMU data

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Abstract

Ill-conditioning of the power flow, reflected in high sensitivity of bus voltage magnitudes to load variation, often serves as a basic indicator of vulnerability to voltage instability. This sensitivity may be quantified by the induced 2-norm associated with the power flow Jacobian inverse, as may be computed from Singular Value Decomposition(SVD). While this conceptual picture is simple, near-real-time assembly of the power flow Jacobian under volatile operating conditions, and monitoring its singular values, presents severe computational and data management problems. As an alternative, work here will propose a "model-free" voltage stability and conditioning monitor, based solely on phasor measurement unit (PMU) data, arguing that such an approach is much better suited to near-real-time application. We first review singular value analysis in voltage stability assessment, and relate these existing approaches to our proposed method. We offer computational evidence that the proposed measure closely tracks the largest singular value of the inverse of the power flow Jacobian under continuous change in operating point, without the need for any network admittance data nor construction of the power flow model. This work goes on to examine characteristics of the PMU-based SVD measure in scenarios of discontinuous changes to network topology. We demonstrate that the new measure also provides a highly sensitive indicator of the occurrence of topology change, that may offer a valuable "consistency check" to supplement directly reported breaker status.

Introduction

In most grid operations centers, monitoring of voltage stability is closely tied to state estimator, and hence may typically be updated at intervals on the order of five minutes. In alert conditions for which the operating point may be changing rapidly, equipment outages may further degrade the system stability margin, and situational awareness may be lost between estimator updates. The state estimator heavily depends on the network topology, which can be changed by unexpected events. The dependence on knowledge of accurate network parameter values and topology can be regarded as a limitation of many current voltage stability monitoring algorithms. These concerns may grow as high penetrations of renewable energy resources in the grid give rise to greater volatility of system operating point. There exists strong need for wide area monitoring on a fast time scale. Due to advanced communication technology and Global Positioning System (GPS), it is possible for Synchrophasor Measurements/Phasor Measurement Units(PMUs) to measure the sinusoidal voltages, currents and powers at each bus and report each of them in the phasor form on a fast time scale. The widespread deployment of such sensors suggests opportunity to supplement state estimator based stability monitoring and system control schemes with measurement-only based measures [1] [2].

Voltage stability refers to the ability of a power system to sustain voltages at all buses after small or large disturbances. Since the time frame of voltage stability varies from a few seconds to many tens of minutes, it is possible to categorize them into a short-term voltage stability and long-term voltage stability [3]. Most research work in short-term voltage stability has been based on dynamic modeling, but a model-free short-term voltage stability monitoring from PMU data has also been reported in [4], which makes a use of Lyapunov exponent for its estimation. A model-based long-term voltage stability detection from PMU data is presented in [5], and another method based on Thevenin impedance matching condition appears in [6], which require prior knowledge of system topology for voltge stability assessments. Monitoring of reactive power margins at generators represents an existing approach to a measurement-only, "model-free" indicator for long-term voltage stability [7], in contrast to a wide variety of techniques that require relatively complete system models [5] [6] [8] [9]. The work to be presented here may be viewed as an evolution of this model-free approach, seeking to make use of the broader class of data available from PMU's.

The conceptual basis of the method to be proposed here is a sensitivity analysis of power flow equation. Eigenvalue or singular value analysis of Jacobian matrix has long been utilized to long-term voltage stability, which seeks to provide indices of system vulnerability to the system operator [10] - [14]. However, such indices have traditionally been computationally expensive, and very dependent upon accurate knowledge of system parameters and topology information, in order to be able to construct the models to be analyzed. These are very demanding tasks in near real-time. The proposed method bypasses the need for parameter data to construct a system model, relying instead on direct measurement of system behavior by means of PMUs. It offers a real-time tool for

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identification of disturbances that may impact voltage stability, such as line switching or outage events. As system operating points become increasingly volatile in modern power systems, it is likely that such real-time measures will become more valuable. In such operating conditions, PMU-based methods to detect topology change effects and outage locations may become particularly useful.

This paper is organized as follows. Singular Value Decomposition(SVD) is reviewed in section II, and a discussion of voltage stability assessment using Jacobian matrix from power flow equation is shown in section III. The proposed algorithm is presented in section IV, followed by its application for voltage stability assessments and topology change detection with simulation results in section V. Conclusion and further direction of this research are shown in section VI.

Singular Value Decomposition

Singular value decomposition (SVD) is a mathematical tool for analysis of matrix structure and characteristic [15]. It decomposes a matrix into a multiplication of three matrices, which are composed of left singular vectors, singular values, and right singular vectors. (1) In case of m-by-n ($m \leq n$) real valued matrix A of rank k, U is an unitary m-by-m matrix whose column vectors are called as left singular vectors. Σ is m-by-n matrix whose upper-left k-by-k sub-block has only diagonal entries termed singular values, and the others are zero. Also, V^T is an unitary n-by-n matrix whose rows are called as right singular vectors.

$$A = U\Sigma V^T \tag{1}$$

As is widely recognized, SVD provides very valuable geometric intuition of the matrix as a linear operator. The singular values may be viewed as identifying the "gain" of the linear operator, acting on orthogonal axes in the domain ("input"), determined by the right singular vectors, and reflected in the range ("output") along the orthogonal axes determined by the left singular vectors. In other words, A maps a unit sphere in n dimension space to a ellipsoid in k dimension space with the directions indicated by left singular vectors and magnitudes of singular values. This mapping is shown for $n = 3$, $m = k = 2$ in Fig. 1

Because of its simplicity, SVD has been widely used in various fields such as linear least square optimization, data compression with reduced rank approximation, neuroscience, computer graphics, and information retrieval from "Big" data. [15] [18] Especially, SVD provides the direction to reduce the high dimensions of data to the lower dimensional space and it often evinces hidden and simplified structure in large data set. It is worth to mention that this characteristic is useful to identify appropriate time window in the proposed algorithm.

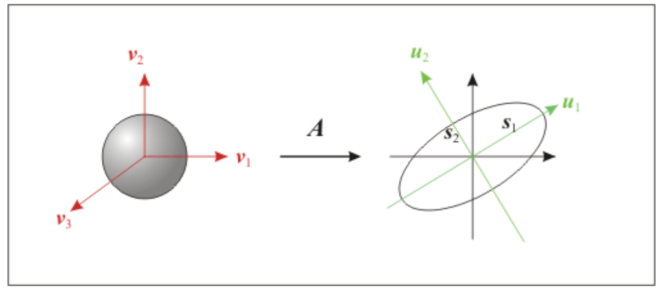

Fig. 1. Geometric interpretation of SVD mapping [17]

In the case of a square matrix, the smallest singular value offers a measure of the distance between the matrix and the nearest singular matrix. For a square, nonsingular matrix, dyad expansion in (2) offers another perspective on the role of the smallest singular value of the original matrix, as setting the maximum gain (i.e., the induced Euclidean norm) for the inverse [16]. This characteristic should also be noted since it is the fundamental idea used in long-term voltage stability assessment with power flow Jacobian.

$$A^{-1} = V\Sigma^{-1}U^T = \sum_{i=1}^{n} \frac{u_i^T v_i}{s_{ii}} \tag{2}$$

Voltage Stability Assessment

The voltage stability assessment using power flow Jacobian has been studied for a long time and the high sensitivity of bus voltage magnitudes to load variation often serve as a basic indicator of proximity of voltage collapse [10] - [14]. Since it is based on linearization method at a operating point, there exist limitations for accounting the power system, which is a nonlinear system. However, they may have been proved its usefulness in terms of investigation of unstable system behavior initiated by large disturbances [5].

$$P_i = \sum_{k=1}^{N} |V_i||V_k|(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}) \tag{3}$$

$$Q_i = \sum_{k=1}^{N} |V_i||V_k|(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}) \tag{4}$$

Equation (3) and (4) show power flow equations, which define the instantaneous operating points for electric power grid in steady states. The equations originate from basic, linear Kirchhoff Current Law (KCL), but they become nonlinear when the current balance constraints are changed to power balance constraints, and power generations and load demands are modeled as fixed power injections or withdrawals from the network. In quasi-steady state, the time variation of power generations and withdrawals drive the entire system: bus voltages evolve in response to these changes. Provided the quasi-static, near-equilibrium assumption remains valid, the power system can be viewed abstractly as a power flow solution engine, taking power injection variations as inputs, and "computing" bus voltage phasors as outputs. A key premise in our work emerges from the viewpoint of that the vector of driving inputs contains both a large signal components, that slowly move the operating point, and small-signal, randomly varying components, that persistently excite the system about its operating point. In mathematical formulation, the power system loads are decomposed into a slowly varying, deterministic process, and faster time scale stochastic process [19]. The deterministic process represents the daily cycle of the load, which is typically high in early afternoon and low in the late evening and night. For the fast time scale stochastic variation in load, early studies tended to adopt simple filtered white noise representations [19]; some recent studies have adopted more sophisticated Ornstein-Uhlenbeck Process models [20]. These may be represented in the form (5),(6) [21], [22].

$$D_t = m_t + X_t \tag{5}$$

$$dD_t = dm_t + \theta(m_t - X_t)dt + \sigma dB_t \tag{6}$$

where B represents brownian motion, and θ and σ denote the mean reversion rate and volatility.

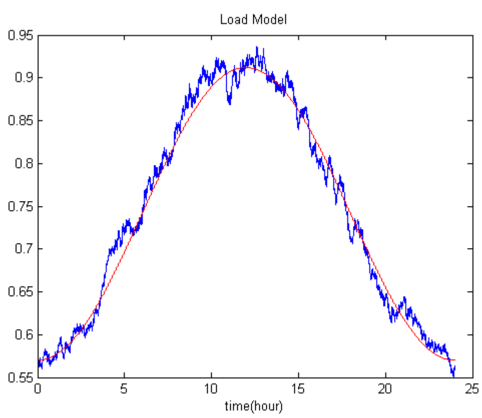

Fig. 2. Representative Sample Path for 24-hour Load Cycle with Additive Ornstein-Uhlenbeck Process. The smooth line in the middle represents the deterministic process.

Our goal will be to consider the impact of such random load variations as driving terms in power balance equations for the electric grid. The "forward" power flow equations, linearized about an operating point can be written as shown in (7). Viewing the physical power system as a power flow solver, it is useful to invert this forward form, treating loads and power injections as inputs, and the output response being phasor angles and voltage magnitudes (as measured by PMUs). Rearranging equation (7) leads us to equation (8), which provides the desired input-output relationship. If the smallest s_{ii} approaches zero, small magnitude variations in power have the potential to yield very large response in bus voltage magnitude and angle variations; as noted earlier, such high sensitivity behavior is recognized as a precursor to voltage instability problems. Indeed, the smallest singular value of Jacobian matrix has been specifically proposed as an index of vulnerability voltage collapse [10].

$$\begin{bmatrix} \Delta \underline{P} \\ \Delta \underline{Q} \end{bmatrix} = \begin{bmatrix} \frac{\partial \underline{P}(\underline{\delta},|\underline{V}|)}{\partial \underline{\delta}} & \frac{\partial \underline{P}(\underline{\delta},|\underline{V}|)}{\partial |\underline{V}|} \\ \frac{\partial \underline{Q}(\underline{\delta},|\underline{V}|)}{\partial \underline{\delta}} & \frac{\partial \underline{Q}(\underline{\delta},|\underline{V}|)}{\partial |\underline{V}|} \end{bmatrix} \begin{bmatrix} \Delta \underline{\delta} \\ \Delta |\underline{V}| \end{bmatrix} \tag{7}$$

$$\begin{bmatrix} \Delta \underline{\delta} \\ \Delta |\underline{V}| \end{bmatrix} = J^{-1} \begin{bmatrix} \Delta \underline{P} \\ \Delta \underline{Q} \end{bmatrix} = \sum_{i=1}^{n} \frac{\underline{u}_i^T \underline{v}_i}{s_{ii}} \begin{bmatrix} \Delta \underline{P} \\ \Delta \underline{Q} \end{bmatrix} \tag{8}$$

Figure 3 is intended to aid visualization of the input-output relationship. The upper figure displays sampled variations in active and reactive loads at a particular bus, applied around a nominal operating point in a 14 bus test power system model. Repeatedly solving the power flow in this model, these input variations are then mapped into corresponding points in the output domain, that of bus voltage magnitude and angle (the full vector of output variations may have many non-zero components; only a two dimensional subspace is displayed here). Because we are solving the power flow, the linear mapping that approximates this relationship is that of the inverse power flow Jacobian. The geometry of the output points closely approximates what one expects from the linear case: the sphere of points in the input space maps to an ellipse of points in the output space. This matches the standard geometric interpretation of the singular values: largest singular value sets the length of the major axis of the "output ellipse," smallest singular value sets the length of the minor axis.

Proposed Algorithm

The fundamental goal of the proposed algorithm is to identify ill-conditioned operating conditions, which may serve as a indicator of vulnerability to voltage collapse. Since proposed algorithm seeks a model-free analysis, the information assumed available will be limited to sampled measurements of voltage magnitudes and phase angles from the buses in the power system. It is typically true that practical PMU measurement sets may include bus power injections and demands. Optimal use of power measurements will be reserved for future work; here we will focus solely on the use of voltage phase angle and magnitude measurements. Under this assumption on available measurements, the goal of the proposed algorithm is simple: Estimate the major axis of the "output ellipse" through use of only measurement information, and track how the quantity evolves in response to load variations or network topology changes. The algorithm employed to estimate the major axis is quite simple, and is closely analogous to the use of SVD tools in other streaming data applications [23]:

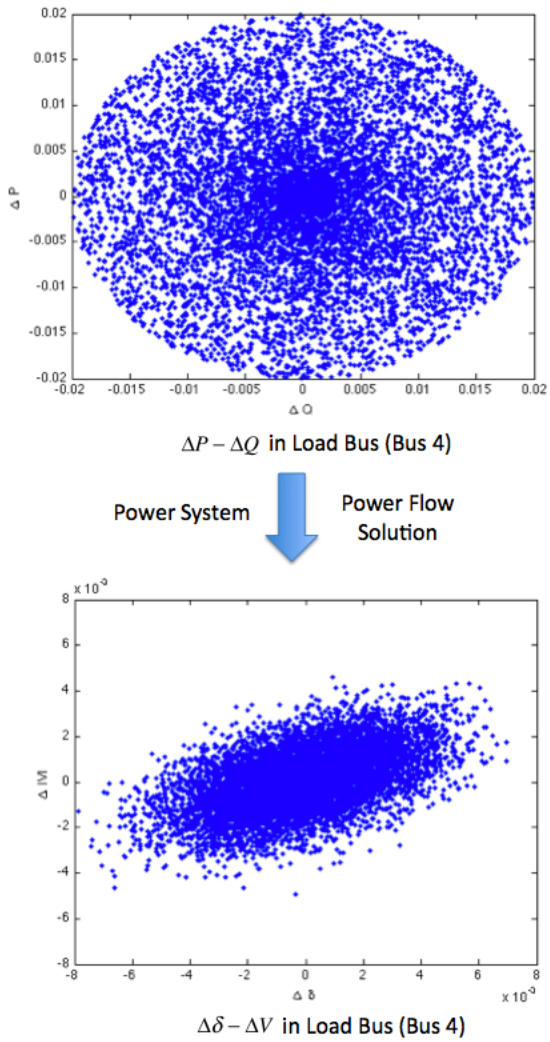

Fig. 3. The graphical description of Power System near a certain operating point. (IEEE 14 bus system used)

after subtracting a base state from the measured PMU data, and possible filtering/bad-data-correction, construct a sliding windowed array of the streaming data, up to the most recent measurement. For this array, one computes the largest (or several largest) singular value(s) and left singular vector(s).

$$\left[\begin{array}{cccccccc} \Delta\bar{\theta}_1 & \Delta\bar{\theta}_2 & \cdots\ \cdots & \Delta\bar{\theta}_L & \Delta\bar{\theta}_{L+1} & \cdots & \Delta\bar{\theta}_k & \cdots \\ \Delta|\bar{V}|_1 & \Delta|\bar{V}|_2 & \cdots\ \cdots & \Delta|\bar{V}|_L & \Delta|\bar{V}|_{L+1} & \cdots & \Delta|\bar{V}|_k & \cdots \end{array}\right]$$

PMU data Matrix

Fig. 4. The graphical description of the algorithm. θ and $|V|$ stand for bus voltage angles and bus voltage magnitudes of buses, measured by PMUs

The base state can be thought of the reference level of monitoring the power system. For estimating power-flow conditioning, the base state is chosen as a "low stress" operating point; typically a lowest normal loading condition. It allows us to monitor how much the power system deviates from the reference level. For better results, the base state may be chosen to be the operating point associated with the predicted "load cycle." The base state subtraction also lessens the impact of different units and off-set between voltage magnitude and phase angle. Roughly speaking, voltage magnitude usually "centers" around 1 as measured in per unit, while phase angle "centers" about 0 in units of radians. Clearly, the subtraction (approximately) removes the offset.

Use of a properly sized time window for the PMU data matrix has significant impact on the result. Recognizing that sampling/reporting rates may vary in different PMU implementations, the time window should be considered in two different aspects: One in terms of time period, and the other in terms of number of samples. For this particular study, the sampling/reporting time in PMU is assumed fixed, and time period or number of samples will be determined if one of them is defined. In considering the appropriate size for the time window of the PMU data matrix, there are trade-offs to be considered. Algebraically, a large window size would seem preferable, to provide dependable estimation of major axis. However, if the window is so large that the system can experience significant change in operating point over the time of the window, the underlying assumption of capturing linearized, "small signal" behavior comes into question. Moreover, a long window clearly imposes a higher computational cost for SVD evaluation, due to the larger matrix size. For small window size, one may view the SVD result as delivering a more recent status of the system with low computational cost, but it can not be as accurate as the large window size. This trade-off is illustrated in Fig. 5, which shows how the largest singular value (LSV) changes as time window increases. The vertical line indicates when network topology changes and solid line is the LSV of inverse Jacobian. If the step change/surge of the measure is defined as "signal" indicative of a topology change event, and the measure in normal operation is defined as "noise", then larger time windows have higher "Signal-to-Noise Ratio" relative to smaller time windows. It should be noted that the LSV of PMU data array increases as time-window increases but increments of the LSV are substantially reduced even though the time-window increases linearly. This implies that the additional samples in the PMU data array may not have significant impact on the estimation compared to the others after some samples are already accumulated in the data array.

It is also supported by the singular vector analysis. Suppose one wishes to characterize the number of components of a singular vector that are significantly different from zero; we will term this the "main dimension." In particular, consider a main dimension defined as the number of component required for sum of squares(S.O.S), 2-norm squares, of normalized

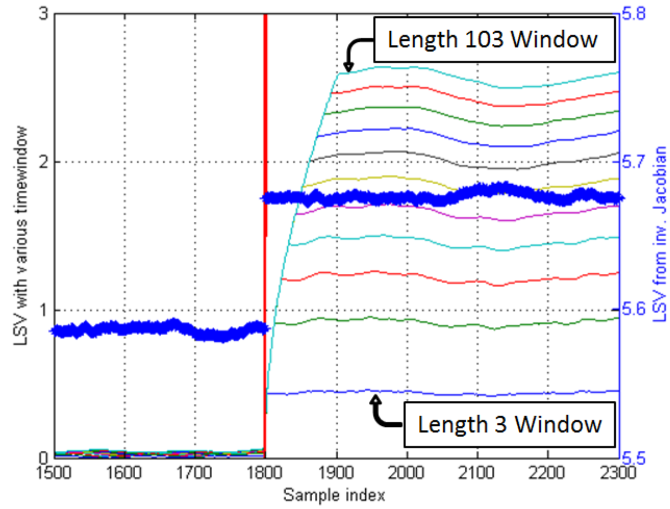

Fig. 5. Effects of Timewindow variation on LV of PMU data array in 30 Hz sampling rate

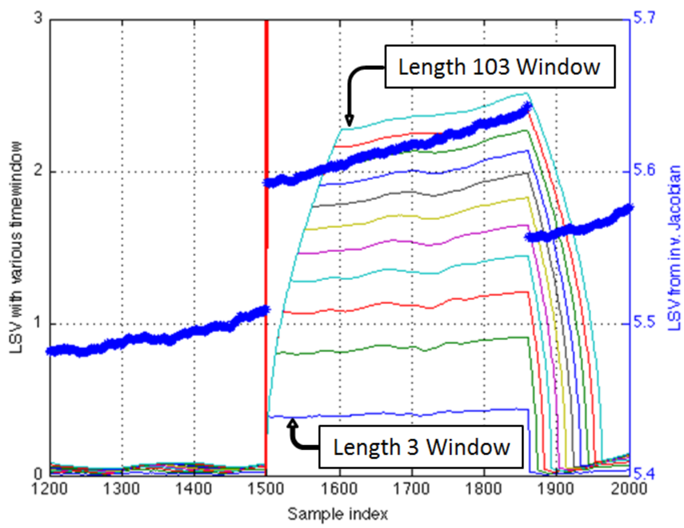

Fig. 6. Effects of Timewindow variation on LV of PMU data array in 0.1 Hz sampling rate

singular vector to be greater than 0.9. For the cases examined here, the main dimension of the singular vector associated with the LSV of PMU data array proves to be relatively small relative to the overall dimension of the vector; i.e., many components are essentially zero. In table I, mean of main dimension is computed with 8640 PMU data samples from IEEE 14, 118, and 300 Bus system.

Figure 7, 8, and 9 show another aspect of this phenomena. Rank of a matrix can be defined as the number of singular values above a certain threshold. Figure 7, 8, and 9 show rank of the PMU data array in IEEE 14, 118, and 300 bus system with various thresholds. The ranks are saturated after they reach a certain number. Although rank with small thresholds tend to result in higher rank of the array and large rank thresholds require wide time window to saturate, there exists a property such that matrix rank stays at certain number even with large timewindow. It supports the fact that any additional samples may not have significant information to estimate the voltage stability after a certain threshold, and the saturation point is reached approximately when the number of timewindow is equal to the number of row of PMU data array. For example, there are 26/234/598 state variables(except slack bus and two variables from each bus) in 14/118/300 Bus system and the rank is staurated approximately when the number of sample is 26/234/598.

TABLE I
MAIN DIMENSION OF SINGULAR VECTOR ASSOCIATED WITH THE LARGEST SINGULAR VECTOR IN IEEE 14, 118, AND 300 BUS SYSTEM.

Bus Type	Data Type	Mean of Main Dim. (S.O.S>0.9)	Total Dim.
14 Bus	Full PMU	12.60	26
	Vmag PMU	6.77	13
	Angle PMU	7.10	13
118 Bus	Full PMU	62.63	234
	Vmag PMU	28.23	117
	Angle PMU	61.93	117
300 Bus	Full PMU	236.00	598
	Vmag PMU	50.30	299
	Angle PMU	236.00	299

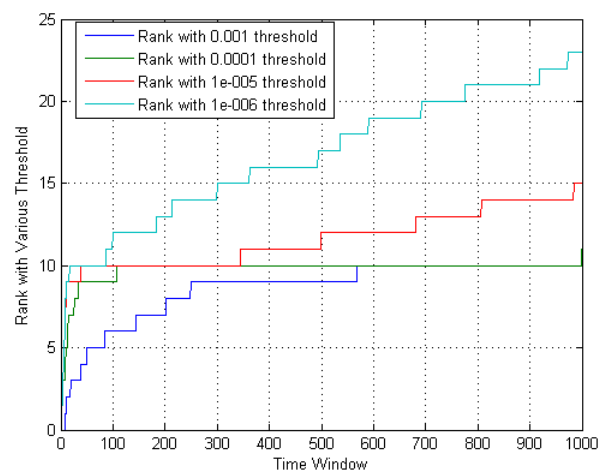

Fig. 7. Rank with various threshold VS. Time window in IEEE 14 Bus

Since voltage stability index from the proposed algorithm should be calculated in near time, low computational cost for SVD computation is essential. The best algorithms for full SVD computation of m-by-n matrix is $O(nm^2 + n^3)$ [16]. The orthogonalization/factorization based algorithm is implemented in LAPACK [16], [24]. However, iterative method for computing SVD such as Lanczos algorithm and Arnoldi's algorithm can substantially reduce its computation time if a small number of the largest singular values (or eigenvalues) and singular vectors (or eigenvectors) in an array is of our main concern. Lanczos algorithm is originated from power-method

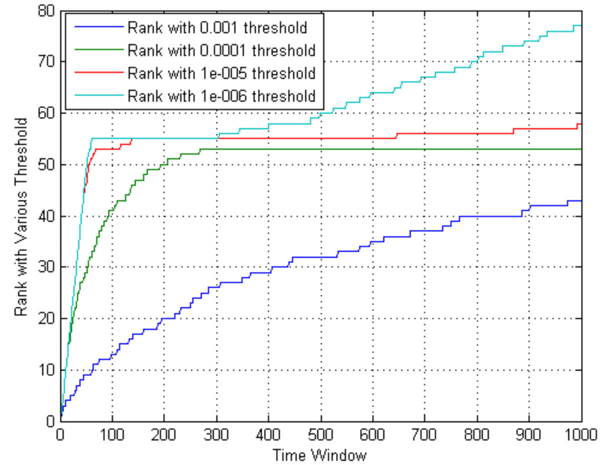

Fig. 8. Rank with various threshold VS. Time window in IEEE 118 Bus

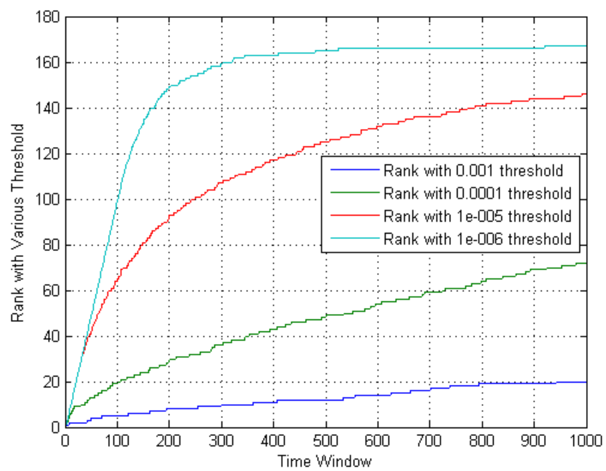

Fig. 9. Rank with various threshold VS. Time window in IEEE 300 Bus

to find eigenvalues and eigenvectors of a square matrix or singular values and singular vectors of a rectangular matrix. The power method for computing eigenvalues/vectors of n-by-n B can be summarized as follows: 1) Start with random vector $\underline{x}_0 \in \Re^n$. 2) Compute $\underline{x}_{i+1} = B\underline{x}_i$ 3) Iterate step 2) until $\underline{x}_{i+1} - \underline{x}_i < tolerance$. Then $\frac{\underline{x}_{i+1}}{||\underline{x}_i||}$ is the normalized eigenvector corresponding to the largest eigenvalue, which is 2-norm of $B\frac{\underline{x}_{i+1}}{||\underline{x}_i||}$. For next largest eigenvalue and eigenvector, go back to step 1) but start with any $\underline{y}_0 \in \Re^n$ that is orthogonal to previous eigenvector. For singular value/vector of a rectangular matrix, A, use the fact that AA^T, A^TA are square matrices. Lanzcos algorithm and Arnoldi's algorithm save the vector $\underline{x}_i$ in step 2) for Gram-Schmidt process to re-orthogonalize them into a basis spanning Kylov subspace. They are implemented in ARPACK [25], [26], [27].

The lanczos algorithm is a little bit slower than the orthogonalization/factorization algorithm for full SVD on the relatively small size of matrix. However, as the matrix size increases, the power method based algorithm outperforms the the orthogonalization/factorization based algorithm as shown in table II. It shows the computation time required to compute a series of the singular values from 8640 PMU data. "svds"(Matlab function) is based on the lanczos algorithm in ARPACK, and "svd"(Matlab function) uses the orthogonalization/factorization algorithm in LAPACK. The third column in the table II shows the case of computing only the largest singular value/vector with power-method.

TABLE II
COMPARISON OF SVD COMPUTATION TIME FOR PROCESSING 8640 PMU DATA SAMPLES IN THE DIFFERENT ALGORITHMS. (SEC)

System	CPU/RAM	6 SVDs svds	Full SVD svd	Power Method Programmed
IEEE 14	Core 2/4GB	49.13	5.82	1.25
	i5/6GB	34.937	2.74	0.7
IEEE 118	Core 2/4GB	164.87	74.94	31.72
	i5/6GB	97.72	38.869	9.092
IEEE 300	Core 2/4GB	298.597	415.33	225.092
	i5/6GB	146.6	208.434	83.476

For computing a series of a small number of largest singular values from arrays sequentially updated by a rank-one modification, even lower computation time can be achieved. The author in [28] compares the SVD updating with rank-one modification to Lanczos algorithm. It shows that the scheme in [28] is about 10 times faster than Lanczos algorithm in low rank 1000-by-1000 matrix and even better performance for low rank 3000-by-3000 matrix. This scheme can be applied to our method since the PMU data array is typically low rank as shown in Fig. 7, 8, 9.

Numerical Study Results

Proposed Method for Voltage Stability assessment

As initial empirical evidence of the performance of this measure, time-series power flow cases were computed for a 24-hour load cycle. As mentioned, for estimating power-flow conditioning, the base state is chosen as a "low stress" operating point; typically a lowest normal loading condition. For the first test, we compute the largest singular value of the inverse Jacobian for each operating point in the series, and the largest singular value of the (pseudo-) PMU data array as described in previous section. A two-degree-of-freedom fit is performed, to allow for difference in offset and scaling. A typical result is shown in Fig. 10, 11, and 12 for IEEE 14, 118, and 300 Bus system.

Figure 10, 11, and 12 show that the proposed algorithm is able to assess power flow conditioning only with the measurements. While larger networks may show slight degradation of quality of fit, experience to date has shown very good fits.

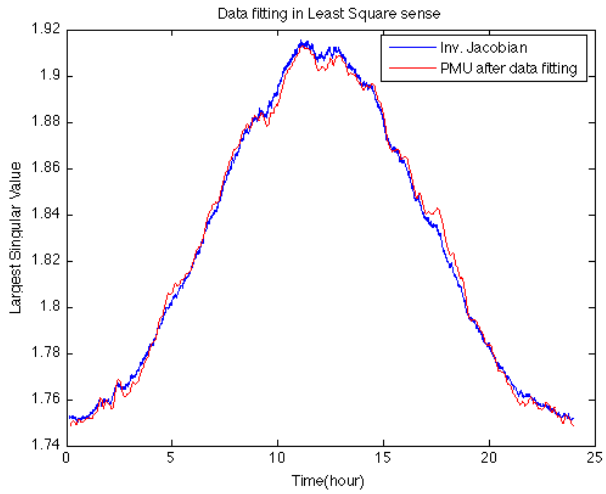

Fig. 10. Largest singular value comparison between Jacobian Inverse and PMU data in IEEE 14 Bus

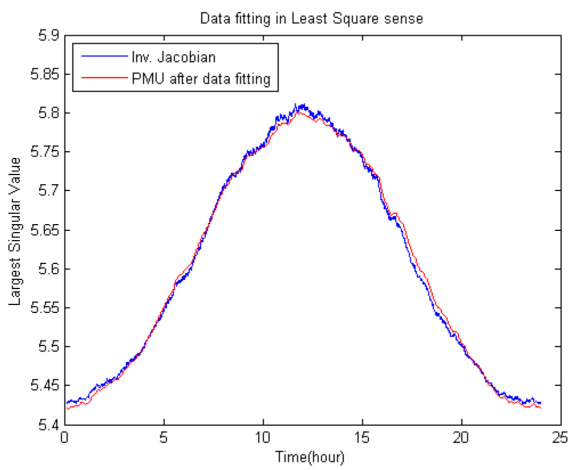

Fig. 11. Largest singular value comparison between Jacobian Inverse and PMU data in IEEE 118 Bus

Proposed Method for Topology Change Detection

As mentioned, the proposed method is also used to detect network topology change by setting the base state to be the operating point based on the predicted "load cycle". Hence, the differences computed to build the array for SVD computation may be interpreted as the deviation of measured quantities away from the power flow solution corresponding to the load cycle prediction, with the predicted network topology. This suggests that when actual measurements result from a physical topology different from that predicted, large jumps away from the base case may be observed. It should be noted that only voltage magnitude part of PMU data is used to construct PMU data array for topology change detection. The voltage magnitude are more sensitive to topology change than phase angle because they controlled in generator bus as long

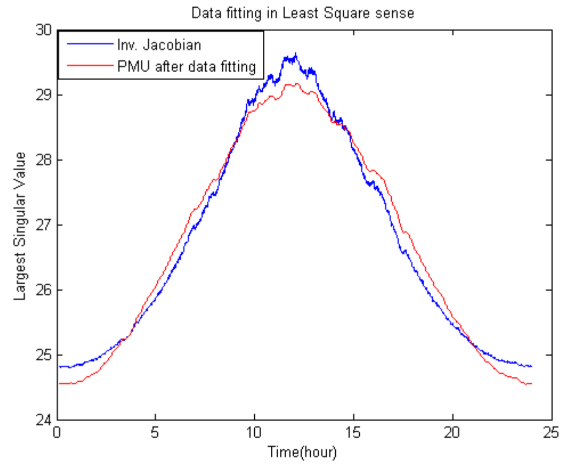

Fig. 12. Largest singular value comparison between Jacobian Inverse and PMU data in IEEE 300 Bus

as the reactive power injections/withdrawals are within their limits and they does not usually change abruptly in normal opearating condition.

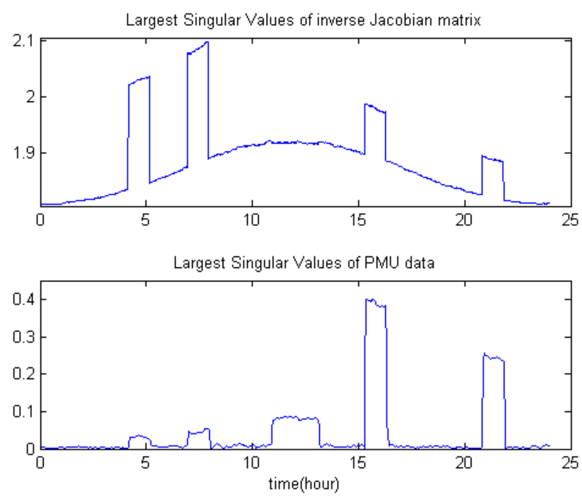

Fig. 13. Time Series of the largest singular value of Jacobian Inverse and PMU data in IEEE 14 Bus

Figure 13 again compares the largest singular value of the exactly computed Jacobian inverse, versus the "measurement-only" PMU-based SVD measure. Note that the topology change events clearly appear as step changes in the inverse Jacobian's largest singular value. Given the nature of the base case, it is worthwhile to emphasize that the PMU-based SVD measure will typically increase for any change, even as the inverse Jacobian's singular value may dip (observe behavior in the interval from hour 11 to hour 13). The reason for the first two events are relatively small to the others is that the change incident happens between bus 2 and 5 and bus 2 is generator bus and bus 5 is surrounded by generator buses. Therefore, the voltage magnitude in bus 2 and 5 are maintained and change a little even with the topology change and results in a small step change in SVD measure. However, this is not typical case for practical system and the SVD measure more clearly captures the topology change in larger system. Also, it should be noted that the “signals” for first two events are 10-15 times larger than normal operation measure even though it looks small in the Fig. 13.

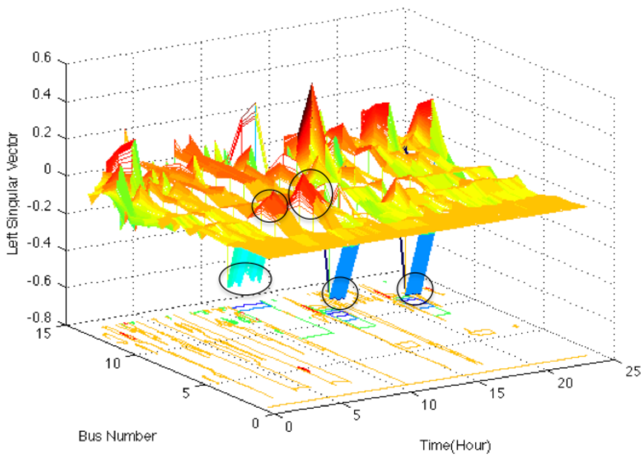

Fig. 14. Contour (Bottom) and 3D Plot for Time Series of the left singular vector from voltage magnitude parts of PMU data in IEEE 14 Bus

Figure 14 examines the corresponding left singular vector, and shows that there are corresponding sudden surges or dips with topology changes. It is true that there are some peaks for a short amount time even in normal operation, but they can be filtered by the information from the largest singular value. The areas highlighted by circles in Fig. 14 suggest that the left singular vector captures information regarding the buses most affected by the topology change events, correctly indicating that the study case had changes incident on bus 5 for the first two events, bus 14 for the 3rd event, and bus 11 for last two events.

Similar test is performed in IEEE 118 Bus system and the results are shown in Fig 15. The proposed measure also successfully captures the changes in the system. It should be noted that the measure is more sensitive to the change than the inverse Jacobian’s singular value.

Also, the left singular from voltage magnitude parts is examined for locating the topology changes. It is worth to note that the left singular vector represent the direction of major axis of “output ellipse”, and it shows which buses are contributing the largest singular value. Therefore, it represents which bus is more vulnerable to the load variation in normal operation. From Fig. 16, it is possible to locate not only where the network topology change is occurred (highlighted by circle) but also where the vulnerable bus is (Bus 44 and 45 relatively stand up in normal operation).

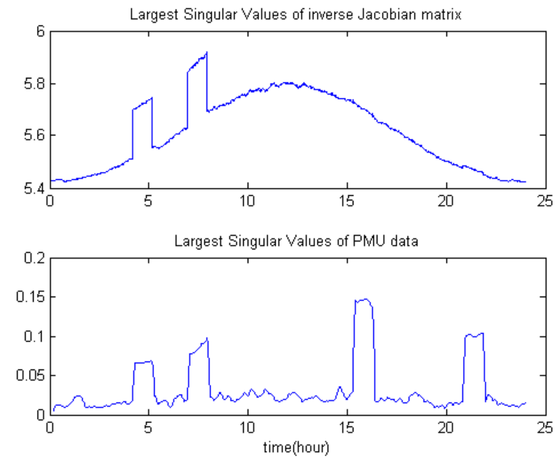

Fig. 15. Time Series of the largest singular value of Jacobian Inverse and PMU data in IEEE 118 Bus

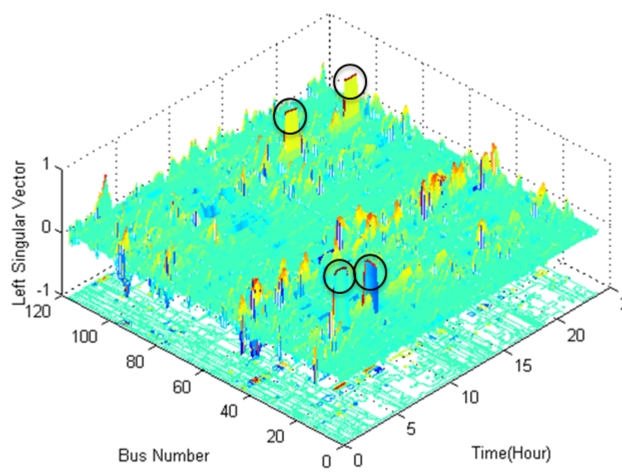

Fig. 16. Contour (Bottom) and 3D Plot for Time Series of the left singular vector from voltage magnitude parts of PMU data in IEEE 118 Bus

Figure 17 and 18 clearly indicate that bus 23 mostly contributes to the largest singular value in the first event and bus 71 for the last event, which implies topology errors occur in the buses.

In IEEE 300 bus system, the proposed measure is able to detect the topology changes even though they are not shown in the largest singular value from Jacobian inverse. Figure 19 shows that there are 4 line incidents for 24 hours and the proposed measure sucessfully identifies them but the index from Jacobian inverse cannot. Similar to two previous cases, the time series of the left singular vector associated with the largest singular value are shown in Fig. 20, 21, and 22. In figure 21, there are 3 buses most affected by the first incident, which are bus 96, 97 and 245. Magnitude of the left singular vector for bus 96 and 245 surge upwards and bus 97 dips down. The actual line incident happens between bus 96 and 97, but

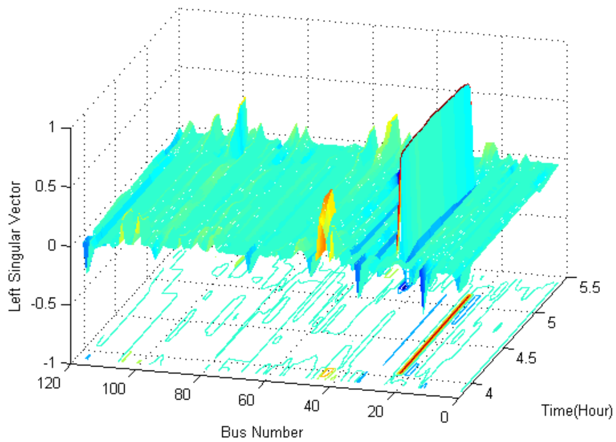

Fig. 17. Parts of 3D Plot for Time Series of the left singular vector from voltage magnitude parts of PMU data in IEEE 118 Bus

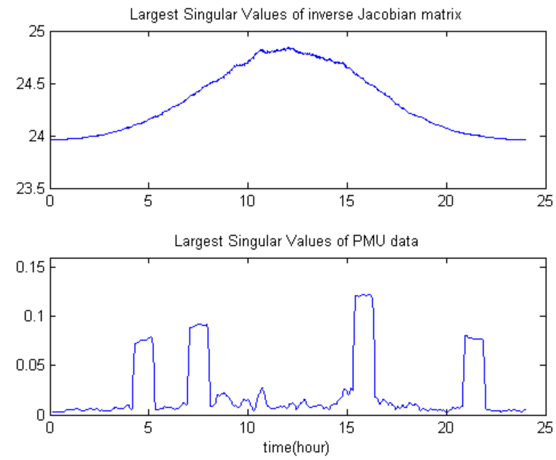

Fig. 19. Time Series of the largest singular value of Jacobian Inverse and PMU data in IEEE 300 Bus

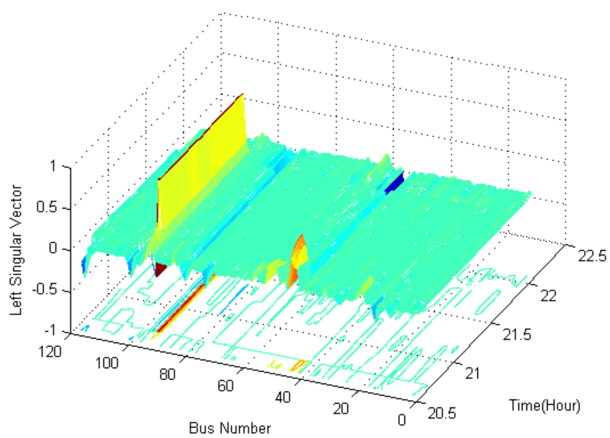

Fig. 18. Parts of 3D Plot for Time Series of the left singular vector from voltage magnitude parts of PMU data in IEEE 118 Bus

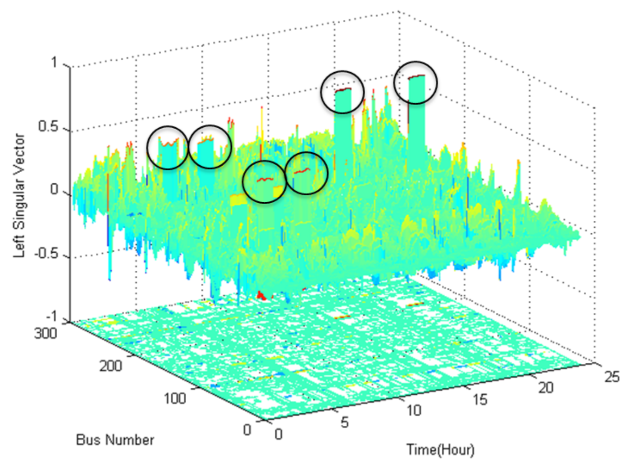

Fig. 20. Contour (Bottom) and 3D Plot for Time Series of the left singular vector from voltage magnitude parts of PMU data in IEEE 300 Bus

the bus 245 also stands up because it is directly connected to bus 97.

Conclusion

Many of trends today, from markets to intermittent renewable integration, suggest the operating conditions in electric power networks are becoming more volatile. This suggests a need for tools to tracking system stability margins and identifying topology errors on time scales faster than state estimator updates. The algorithm presented in this paper seeks to serve this need through a model-free voltage stability indicator that also shows promise as topology change detector in near real time, using only measurement information that is becoming widely available through PMUs. Also, the proposed indicator is able to detect any topology change in the network and locate the events from singular value and vector analysis. This suggests an opportunity to control the system operating point with high resolution data, which is essential for stabilizing volatile system.

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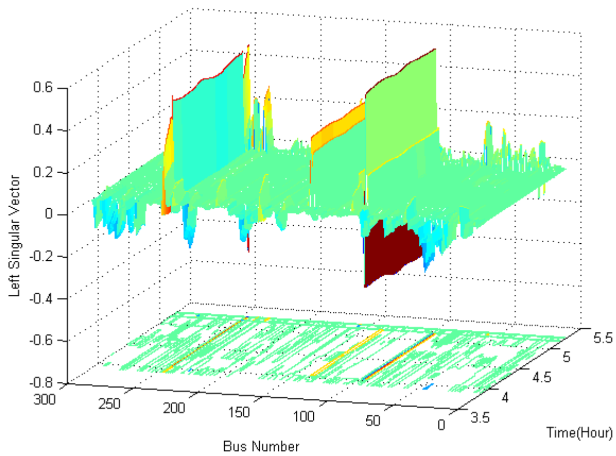

Fig. 21. Parts of 3D Plot for Time Series of the left singular vector from voltage magnitude parts of PMU data in IEEE 300 Bus

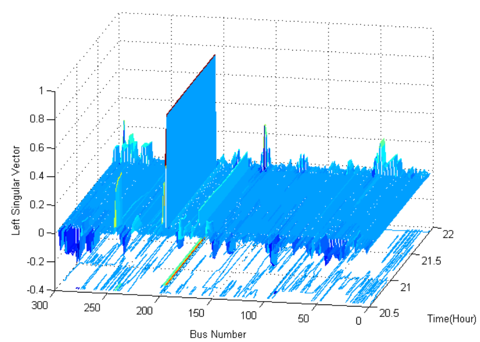

Fig. 22. Parts of 3D Plot for Time Series of the left singular vector from voltage magnitude parts of PMU data in IEEE 300 Bus

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