2013 IREP Symposium-Bulk Power System Dynamics and Control -IX (IREP), August 25-30, 2013, Rethymnon, Greece

An optimization-based approach for assessing the benefits of residential battery storage in conjunction with solar PV

Elizabeth L. Ratnam, Steven R. Weller and Christopher M. Kellett

Abstract— **If residential customers are billed the real-time cost of electricity, battery storage may provide a cost-benefit. Consequently there is an opportunity to reduce the consumer's cost of electricity by maximizing PV energy output and time-shifting load through battery storage. In this paper, we propose a framework based on quadratic programming by which the cost-benefit of energy time-shifting to a given customer may be evaluated. Accordingly, the consumer may justify expenditure on battery storage through either a least cost option of capital investment or choose to utilize existing electric vehicle (EV) battery storage, if available.**

I. Introduction

Well coordinated distributed energy resources (DERs) provide advantages to consumers and utilities alike, including economic savings, increased security of supply, greater network efficiencies and a reduced network burden during peak and minimum load periods [1], [2], [3]. Challenges to optimally coordinating DERs include intermittency [4] and infrastructure constraints [3]. Methods to address coordination challenges of DERs include passive control, e.g demand response pricing [5], active control, by directly reducing energy consumption and/or generation [6], and energy time-shifting through battery storage [4].

For residences with DERs, several authors have considered approaches to energy time-shifting which involve coordinated scheduling of battery storage [5], [7] and [8]. A range of optimization-based approaches to energy time-shifting have been presented, including approaches based on linear programming [9], particle swarm optimization [10], and NP-hard task scheduling [11]. Our approach in this paper is similar in that we seek to optimally schedule battery storage in conjunction with residential solar photovoltaic (PV) systems in such a way that the PV energy output is maximized. Implicit in our approach is the expectation that future costs of residential batteries will reduce significantly, for example wide-spread uptake of electric vehicles (EV) may provide opportunities to utilize decommissioned EV batteries [12], [13].

In this paper we present a quadratic program (QP)-based minimization of the energy supplied by, or to, the grid in a residential PV system with co-located battery storage. Under the assumption that daily residential load and solar PV generation are known (or can be accurately forecast), the QP-based framework presented in this paper leads to a battery charge and discharge schedule for the day ahead. Our approach for finding the preferred rate to charge and discharge a battery is introduced in Section II and the financial incentives to energy time-shift are defined in Section III. Section IV quantifies consumer savings, and Section V presents an assessment of the benefits for residential battery storage for the residential network presented in Figure 1, including guidance on battery selection.

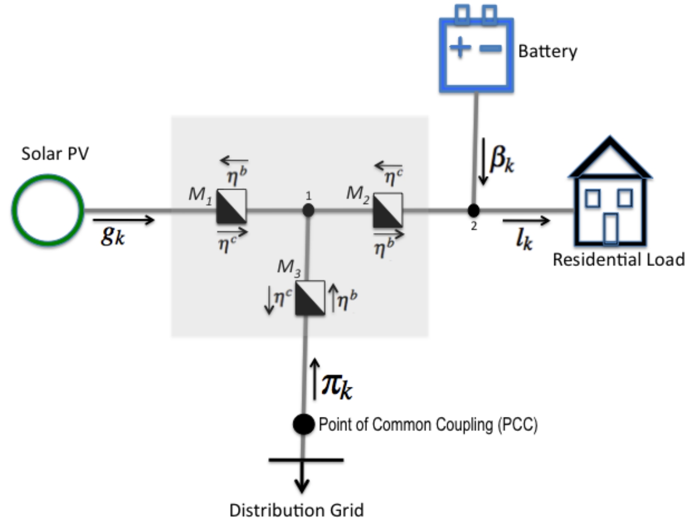

Fig. 1. Residential network illustrating the direction of positive power flows and financial incentives to energy time-shift. Arrows associated with g_k, l_k, β_k and π_k illustrate the assumed direction of positive power flow. Financial incentives for each meter M_1, M_2 and M_3 are represented by vectors η^b and η^c (in \$/kWh), in which arrows illustrate the direction of power flow relevant for η^b and η^c.

Notation

Let $\mathbb{R}^s$ denote s-dimensional vectors of real numbers and $\mathbb{R}^s_{\geq 0}$ s-dimensional vectors with all non-negative components. $\mathbf{I}$ denotes the s-by-s identity matrix, $\mathbf{0}$ denotes the s-by-s all-zero matrix, and $\mathbf{1} \in \mathbb{R}^s_{\geq 0}$ denotes the all-1s column vector of length s.

II. Problem formulation

A. Definitions and objectives

Figure 1 illustrates the topology of the network under consideration, including a set of meters $\mathscr{M} = \{M_1, M_2, M_3\}$ installed for the purpose of billing and compensation. For each $k \in \{1, \ldots, s\}$, meter M_1 measures the average PV generation g_k (in kW), meter M_2 measures the average power from node 1 to node 2 ($l_k - \beta_k$ in kW), and meter M_3 measures the average power π_k (in kW) supplied by (or to) the grid. Meters M_2 and M_3 may be bi-directional, whereas meter M_1 needs only be unidirectional since PV generation $g_k \geq 0$ for all k. Also shown in Figure 1 are vectors η^b and η^c, which represent financial incentives for billing and compensation respectively, defined in Section III.B.

The power flows indicated in Figure 1 are represented by vectors of length s, where s is the number of time intervals of length Δ, and $T = s\Delta$ (in hours) is the time window of interest. In this paper we generally consider $T = 24$ hours, $\Delta = 1/2$ hour (30 minutes), which implies $s = 48$. Other choices are certainly possible, subject only to commensurability of T, Δ, and s.

We represent the average power delivered to the residential load (in kW) over the period $((k-1)\Delta, k\Delta)$ by l_k for all $k \in \{1,\ldots,s\}$, and define the *load profile* over $[0,T]$ as $l := [l_1,\ldots,l_s]^T \in \mathbb{R}^s_{\geq 0}$. Likewise we represent the average PV generation (kW) over the period $((k-1)\Delta, k\Delta)$ by g_k for all $k \in \{1,\ldots,s\}$, and define the *generation profile* over $[0,T]$ as $g := [g_1,\ldots,g_s]^T \in \mathbb{R}^s_{\geq 0}$. In what follows, we assume the load and generation profiles are given.

We represent the average power (kW) delivered from (or to) the battery over the period $((k-1)\Delta, k\Delta)$ by $\beta_k > 0$ (or $\beta_k < 0$) for all $k \in \{1,\ldots,s\}$, and define the *battery profile* over $[0,T]$ as $\beta := [\beta_1,\ldots,\beta_s]^T \in \mathbb{R}^s$. By convention we represent charging (discharging) of the battery by $\beta_k < 0$ ($\beta_k > 0$). To capture the limited "charging/discharging capacity" of the battery, we constrain β_k as follows:

$$\underline{B} \leq \beta_k \leq \overline{B} \qquad \text{for all } k \in \{1,\ldots,s\}, \tag{1}$$

where typically $\underline{B} < 0$ and $\overline{B} > 0$.

Given β, the *state of charge* of the battery (in kWh) at time $k\Delta$ is denoted by χ_k, where

$$\chi_k := \chi_0 - \sum_{j=1}^{k} \beta_j \Delta \qquad \text{for all } k \in \{1,\ldots,s\}, \tag{2}$$

and χ_0 denotes the initial state of charge of the battery. If we represent the battery capacity (in kWh) by $\overline{C} \in \mathbb{R}_{\geq 0}$, it necessarily follows that both the initial state of charge χ_0 and the state of charge at later time $k\Delta$ are constrained as follows:

$$\underline{C} \leq \chi_k \leq \overline{C} \qquad \text{for all } k \in \{0,1,\ldots,s\}, \tag{3}$$

where typically $\underline{C} = 0$.

We represent the average power (in kW) supplied by (or to) the grid over the period $((k-1)\Delta, k\Delta)$ by π_k for all $k \in \{1,\ldots,s\}$ and define the *grid profile* over $[0,T]$ as $\pi := [\pi_1,\ldots,\pi_s]^T \in \mathbb{R}^s$. By convention we represent power flowing from (to) the grid to (from) the energy system by $\pi_k > 0$ ($\pi_k < 0$).

From the configuration of the residential energy system in Figure 1, we observe that the following power balance equation must hold:

$$l_k = \pi_k + g_k + \beta_k, \qquad \text{for all } k \in \{1,\ldots,s\}. \tag{4}$$

We seek to minimize the impact of the residential energy system on the grid, given a financial incentive to energy time-shift, by minimizing the following expression:

$$\sum_{k=1}^{s} h_k \pi_k^2, \tag{5}$$

where h_k is a selectable weighting such that $h_k \geq 1$ for all $k \in \{1,\ldots,s\}$.

Specifically, given load and generation profiles l and g, and given battery constraints χ_0, $\overline{C}$, $\underline{C}$ and $\overline{B}$, $\underline{B}$ we seek a battery profile β and a grid profile π which minimize the expression in (5), subject to satisfaction of the power balance in equation (4).

The minimization in (5) is subject to both inequality and equality constraints imposed by the battery (1)–(3) and the power balance equation in (4), respectively. Lemma 1 below establishes this constrained minimization as a quadratic program (QP). Let

$$\mathbf{T} := \Delta \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 1 & 1 & 0 & \cdots & 0 \\ 1 & 1 & 1 & & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \cdots & 1 \end{bmatrix} \in \mathbb{R}^{s \times s}, \tag{6}$$

$$\overline{\mathbf{B}} := \mathbf{1}\overline{B}, \qquad \underline{\mathbf{B}} := \mathbf{1}\underline{B}, \tag{7}$$

$$\overline{\mathbf{C}} := \mathbf{1}(\chi_0 - \underline{C}), \qquad \underline{\mathbf{C}} := \mathbf{1}(\overline{C} - \chi_0), \tag{8}$$

$$\mathbf{H} := \operatorname{diag}(h_1,\ldots,h_s). \tag{9}$$

Lemma 1: The minimization of expression (5), subject to battery constraints (1)-(3) and the power balance equation (4), can be written as

$$\min_{x \in \mathbb{R}^{2s}} \quad x^T H x \tag{10}$$

such that

$$A_1 x \leq b_1, \tag{11}$$

$$A_2 x = b_2, \tag{12}$$

where

$$x := \begin{bmatrix} \pi^T & \beta^T \end{bmatrix}^T \in \mathbb{R}^{2s}, \tag{13}$$

$$H := \begin{bmatrix} \mathbf{H} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \in \mathbb{R}^{2s \times 2s}, \tag{14}$$

$$A_1 := \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{0} & -\mathbf{I} \\ \mathbf{0} & \mathbf{T} \\ \mathbf{0} & -\mathbf{T} \end{bmatrix} \in \mathbb{R}^{4s \times 2s}, \tag{15}$$

$$A_2 := \begin{bmatrix} \mathbf{I} & \mathbf{I} \end{bmatrix} \in \mathbb{R}^{s \times 2s}, \tag{16}$$

$$b_1 := \begin{bmatrix} \overline{\mathbf{B}}^T & \underline{\mathbf{B}}^T & \overline{\mathbf{C}}^T & \underline{\mathbf{C}}^T \end{bmatrix}^T \in \mathbb{R}^{4s}, \tag{17}$$

$$b_2 := l - g \in \mathbb{R}^{s}. \tag{18}$$

Proof: The result follows directly from equations (1)–(3), and substitution of definitions (6)–(9) into equations (13)–(18). ■

The grid profile resulting from Lemma 1 is said to be *QP energy-shifted*, and we will refer to the process of a customer using Lemma 1 to determine daily battery and grid profiles as QP energy-shifting.

B. Example

We illustrate the application of Lemma 1 at a customer's premise where the load includes a utility controlled heated water cylinder [14].[1] Let $T = 24$ hours, $\Delta = 30$ minutes and $s = T/\Delta = 48$, and let load and generation profiles l and g be specified as shown in Fig. 2(a). Given battery capacities $\overline{C} = 1$kWh and 10kWh, let $\underline{C} = 0$, $\chi_0 = 0.5\,\overline{C}$ (initial battery state of charge), $\overline{B} = -\underline{B} = 1\,$kW (charge/discharge limits), and let weights $h_k = 1$ for all k.

In Fig. 2(a) we observe the load profile peaks around midnight, consistent with the utility switching on the all-electric-heated water cylinder at the customer premises, and the generation profile peaks around midday. Consequently the peak generation does not align with the peak load at the residence.

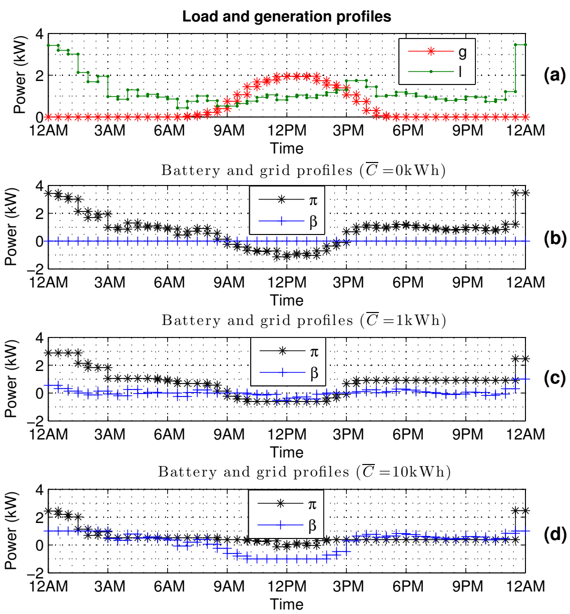

Fig. 2. (a) Load and generation profiles l and g; (b) grid and battery profiles π and β for $\overline{C}$ =0kWh; (c) grid and battery profiles π and β for $\overline{C}$ =1kWh; (d) grid and battery profiles π and β for $\overline{C} = 10$kWh.

Figure 2(b) illustrates the *base-line* grid profile to which we compare the grid profiles in Fig. 2(c)–(d). The base-line grid profile has no battery to energy time-shift the grid profile π appearing in (5) ($\overline{C} = 0$kWh). Therefore π is calculated directly from the power balance equation in (4) since clearly $\beta_k = 0$. The grid profiles π illustrated in Figure 2(c)–(d) arise from the solution of the QP in Lemma 1. Comparing the base-line results in Figure 2(b) to the grid profile in Fig. 2(c), we observe the 1kW battery charges ($\beta_k < 0$) to increase the base-line grid profile (e.g. from -1.13kW to -0.61kW between 11.30-Midday), and discharges ($\beta_k > 0$) to reduce the base-line grid profile (e.g. from 3.46kW to 2.46kW between 23.30-Midnight). In Fig. 2(d) we observe further reductions in the magnitude of π, except between 23.30-Midnight, due to the battery discharge constraint of 1kW.

[1] In some countries, residents often allow the utility to control their all-electric-heated water systems for periods in the day, given a financial incentive. For these customers, the utility switches their water-heating services on during periods of low load, and off during periods of peak-load, in a manner that ensures minimal impact to the network.

This example demonstrates the reductions in magnitude of the grid profile π subject to constraints, including the battery charge/discharge constraints, and capacity $\overline{C}$.

C. Extended definition of grid profile

We now extend our definition of *grid usage* over the period $((k-1)\Delta, k\Delta)$ to include explicit reference to the battery capacity $\overline{C}$, and weights h_k as follows:

$$\pi_k^{\overline{C}}(h_k) := l_k - g_k - \beta_k \qquad \text{for all } k \in \{1, \ldots, s\}, \tag{19}$$

where l_k, g_k, β_k and h_k remain as previously defined. We consequently denote the *grid profile* over $[0, T]$ by

$$\pi^{\overline{C}}(\mathbf{H}) := [\pi_1^{\overline{C}}(h_1), \ldots, \pi_s^{\overline{C}}(h_s)]^T \qquad \in \mathbb{R}^s. \tag{20}$$

When battery capacity $\overline{C} = 0$, it follows

$$\pi_k^0 := l_k - g_k \qquad \text{for all } k \in \{1, \ldots, s\}, \tag{21}$$

since the battery charging/discharging capacity $\beta_k = 0$, $k \in \{1, \ldots, s\}$. The case where $\overline{C} = 0$ is defined as a *base-line* grid profile against which we compare future grid profiles and is denoted by

$$\pi^0 := [\pi_1^0, \ldots, \pi_s^0]^T. \tag{22}$$

Furthermore, we note π^0 is not a function of the selectable weights in $\mathbf{H}$, as the base-line grid profile is solely a function of load and generation profiles in (21).

Remark 1: The grid profile π obtained from solving the quadratic program in Lemma 1 depends not only on the battery constraints $\overline{C}$, $\underline{C}$, $\overline{B}$, $\underline{B}$ and χ_0, selected weightings h_k, but also the load and generation profiles l and g, respectively. Consequently π is the function $\pi = \pi(\,l,\ g,\ \overline{C},\ \underline{C},\ \overline{B},\ \underline{B},\ \chi_0,\ \mathbf{H})$. For notational simplicity, however, we will henceforth omit the functional dependence of π on the the load/generation profiles and all the battery constraints other than the battery capacity $\overline{C}$, preferring instead to simply write $\pi^{\overline{C}}(\mathbf{H})$, where no ambiguity arises. This notational convention reflects our primary degrees of design flexibility, namely battery capacity $\overline{C}$ and the weighting matrix $\mathbf{H}$.

III. Billing for a single customer

In this section we define the *energy bill* for a single residential customer for the household PV system depicted in Figure 1. To calculate the energy bill we require a *financial policy* (in \$/kWh), and a battery of capacity $\overline{C}$ when the customer uses QP energy-shifting (Lemma 1). Since the *financial policy* requires meters in certain locations, with particular modes of operation, we also define the *metering topology* in Section III.A.

A. Metering topology

To formalize the notion of *metering topology* we define two *metering modes* in terms of the meters $M \in \mathscr{M}$, and provide an example with respect to meter M_2 shown in Figure 1.

1. Gross metering mode: requires only uni-directional metering. We say that meter M_2 operates in *gross metering mode* if it measures power flow from node 1 to the battery/load node 2, but not power delivered in the reverse direction. That is, meter M_2 measures and records only power flows for which $l_k - \beta_k \geq 0$. In the event $l_k - \beta_k < 0$, the meter records 0kW.

2. Net metering mode: requires bi-directional metering [15]. We say that meter M_2 operates in *net metering mode* if it measures power flow in both directions, i.e. from node 1 to the battery/load node 2 ($l_k - \beta_k \geq 0$), as well as power delivered in the reverse direction (i.e. $l_k - \beta_k < 0$).

The metering topology is defined by the mode of operation (gross or net) of each meter $M \in \mathscr{M}$ in Figure 1. If in gross metering mode, the direction of power flow must also be included.

The metering topologies considered in this paper are defined below, with the direction of positive power flow as per Figure 1, defined in Section II.A.

- *Metering topology 1*: M_1 and M_2 operate in gross metering mode. M_3 is not installed. M_1 measures and records the generation profile $g_k \geq 0$ for all k, M_2 measures and records the power flow $l_k - \beta_k \geq 0$ for all k.
- *Metering topology 2*: M_3 operates in net metering mode. M_1 and M_2 are not installed.

B. Financial policies

Financial incentives intended to modify consumer behaviour associated with energy usage are captured in our definition of a *financial policy*. Example incentives include *time-of-use* (T.O.U.) pricing, *feed-in-tariffs* and *net metering* [15], [16]. In what follows we formalize the notion of a financial policy, encompassing one or more of the example incentives above, along with other more general cases.

To formalize the notion of a *financial policy*, we define an *electricity billing profile* and an *electricity compensation profile* over $[0,T]$, for each installed meter in $\mathscr{M}$. The direction of power flow associated with electricity billing/compensation is defined with reference to the direction of positive power that is specified at each meter $M \in \mathscr{M}$. We denote *electricity billing* (in \$/kWh) at meter $M \in \mathscr{M}$ over the period $((k-1)\Delta, k\Delta)$ by $\eta_k^b(M)$ for all $k \in \{1,\ldots,s\}$, and define the *electricity billing profile* over $[0,T]$ at M as $\eta^b(M) := [\eta_1^b(M),\ldots,\eta_s^b(M)]^T \in \mathbb{R}_{\geq 0}^s$. Likewise we denote the *electricity compensation* (in \$/kWh) at meter $M \in \mathscr{M}$ over the period $((k-1)\Delta, k\Delta)$ by $\eta_k^c(M)$ for all $k \in \{1,\ldots,s\}$, and define the *electricity compensation profile* over $[0,T]$ at M as $\eta^c(M) := [\eta_1^c(M),\ldots,\eta_s^c(M)]^T \in \mathbb{R}_{\geq 0}^s$.

In order to implement a financial policy, certain types of meters are required in particular locations. For example a financial policy may require the meter M_1 (in Fig. 1), which records positive power flows from the rooftop PV to node 1. For this meter the financial policy will specify the electricity billing and compensation profiles $\eta^b(M_1)$, $\eta^c(M_1)$, respectively. If the electricity billing (or compensation) profile at meter M_1 is defined by $\eta_j^b(M_1) = 0$ (or $\eta_j^c(M_1) = 0$) for all $j,k \in \{1,\ldots,s\}$, than it is sufficient that meter M_1 operates in gross metering mode; where the power flow to be measured is in the same direction specified for electricity compensation (or billing).

We now define a *financial policy* over $[0,T]$, by the day ahead electricity billing and compensation profiles at each installed meter in $\mathscr{M}$. An example financial policy is defined with reference to Fig. 1 for $\mathscr{M} = \{M_1, M_2, M_3\}$. The direction of positive power flow at meter M_1 is defined by g (from the solar PV to node 1), and electricity is compensated in this direction $\eta^c(M_1)$. The direction of positive power flow at meter M_2 is defined by $l - \beta \geq 0$ (from node 1 to node 2), and electricity is billed in this direction $\eta^b(M_2)$. The direction of positive power flow at meter M_3 is defined by π (from the PCC to node 1), and electricity is billed in this direction $\eta^b(M_3)$. For each electricity compensation (or billing) profile $\eta^b(M)$ (or $\eta^c(M)$), there also exists an electricity billing (or compensation) profile $\eta^c(M)$ (or $\eta^b(M)$) for power flowing against the positive direction at meter $M \in \mathscr{M}$.

The financial policies considered in this paper are defined with reference to metering topologies 1 and 2 respectively, defined in Section III.A. The financial policy associated with metering topology 1 includes an electricity compensation profile at meter M_1 (for power flow from the solar PV to node 1), and an electricity billing profile at meter M_1 (for power flows in the reverse direction), represented by $\eta^c(M_1)$ and $\eta^b(M_1)$ respectively; and an electricity compensation profile at meter M_2 (for power flow from node 2 to node 1), and an electricity billing profile at meter M_2 (for power flows from node 1 to node 2), represented by $\eta^c(M_2)$ and $\eta^b(M_2)$ respectively. Furthermore $\eta_j^b(M_1) = 0$ and $\eta_j^c(M_2) = 0$, for all $j,k \in \{1,\ldots,s\}$, hence it is sufficient that meters M_1 and M_2 operate in gross metering mode, as per the definition of metering topology 1. The financial policy associated with metering topology 2 has an electricity compensation profile at meter M_3 (for power flow from node 1 to PCC), and an electricity billing profile at meter M_3 (for power flow from the PCC to node 1), represented by $\eta^c(M_3)$ and $\eta^b(M_3)$ respectively.

C. Energy bill

We now define the residential *energy bill* (in \$/day) in terms of the respective financial policy associated with metering topologies 1 and 2 (Section III.B.). We assume the day ahead billing and compensation profiles in the respective financial policies are fixed by the utility or regulatory body and available to the consumer. For a fixed battery capacity $\overline{C}$, we also observe that the choice of weighting matrix $\mathbf{H}$ is critical to minimizing the energy bill, when the customer uses QP energy-shifting.

In defining the energy bill, we include the cost associated with the battery's initial state of charge χ_0, denoted by η_0^b (in \$/kWh) and the cost associated with the battery's remaining state of charge χ_s, denoted by η_0^c (in \$/kWh). In this paper we assume the cost associated with the battery's initial and final states of charge are equivalent $\eta_0^b = \eta_0^c$, though other choices are possible and investigation of this case is left for future research.

In the formalization of the energy bill associated with the financial policy relating to metering topology 1, we define $\sigma_k(M_1)$ and $\sigma_k(M_2)$ as follows:

$$\sigma_k(M_1) = \begin{cases} \eta_k^c(M_1), & \text{if } g_k \geq 0 \\ \eta_k^b(M_1), & \text{if } g_k < 0, \end{cases} \tag{23}$$

$$\sigma_k(M_2) = \begin{cases} \eta_k^b(M_2), & \text{if } l_k - \beta_k \geq 0 \\ \eta_k^c(M_2), & \text{if } l_k - \beta_k < 0, \end{cases} \tag{24}$$

and denote $\sigma(M_1) := [\sigma_1(M_1), \ldots, \sigma_s(M_1)]^T \in \mathbb{R}^s_{\geq 0}$ and $\sigma(M_2) := [\sigma_1(M_2), \ldots, \sigma_s(M_2)]^T \in \mathbb{R}^s_{\geq 0}$ over the period $[0,T]$. Recall $\eta_j^b(M_1) = 0$ and $\eta_j^c(M_2) = 0$, for all $j,k \in \{1,\ldots,s\}$.

In order to minimize the energy bill associated with metering topology 1, we choose the weighting matrix for a given battery capacity with constraints (1)–(3) known and fixed, as follows:

$$\mathbf{H}_1 := \mathbf{H}(\sigma(M_1), \sigma(M_2)). \tag{25}$$

Through an appropriate selection of $\mathbf{H}_1$ in equation (25), we define the residential energy bill associated with metering topology 1, denoted by $\Sigma^{\overline{C}}(\mathbf{H}_1)$ (in \$/day) by

$$\Sigma^{\overline{C}}(\mathbf{H}_1) := T((l-\beta)^T \sigma(M_2) - g^T \sigma(M_1)) + \eta_0^b(\chi_0 - \chi_s). \tag{26}$$

When the battery capacity $\overline{C} = 0$, the energy bill defined in (26) reduces to

$$\Sigma^0 := T(l^T \eta^b(M_2) - g^T \eta^c(M_1)), \tag{27}$$

since the battery charging/discharging capacity $\beta_k = 0$ for all $k \in \{1,\ldots,s\}$, rendering the selectable weights in $\mathbf{H}_1$ irrelevant. The case where $\overline{C} = 0$ also serves as a *base-line energy bill*, which we use as a comparison when assessing the benefits of battery storage.

Remark 2: The *energy bill* notational convention $\Sigma^{\overline{C}}(\mathbf{H}_1)$ is simplified, and consistent with our primary degrees of design flexibility, the battery capacity $\overline{C}$ and the weighting matrix.

To formalize the energy bill associated with metering topology 2, we define $\sigma_k(M_3)$ in terms of the financial policy as follows:

$$\sigma_k(M_3) = \begin{cases} \eta_k^b(M_3), & \text{if } \pi_k^{\overline{C}}(h_k) \geq 0 \\ \eta_k^c(M_3), & \text{if } \pi_k^{\overline{C}}(h_k) < 0, \end{cases} \tag{28}$$

and we denote $\sigma(M_3) := [\sigma_1(M_3), \ldots, \sigma_s(M_3)]^T \in \mathbb{R}^s_{\geq 0}$ over the period $[0,T]$. In order to minimize the energy bill associated with metering topology 2, we choose the weighting matrix for a given battery capacity with constraints (1)–(3) known and fixed, as follows:

$$\mathbf{H}_2 := \mathbf{H}(\sigma(M_3)). \tag{29}$$

Through appropriate selection of $\mathbf{H}_2$ in (29), we define the energy bill associated with the financial policy relating to metering topology 2 by

$$\Sigma^{\overline{C}}(\mathbf{H}_2) := T\pi^{\overline{C}}(\mathbf{H}_2)^T \sigma(M_3) + \eta_0^b(\chi_0 - \chi_s), \tag{30}$$

which reduces to the *base-line energy bill* for $\overline{C} = 0$ as follow:

$$\Sigma^0 := T(l-g)^T \sigma(M_3), \tag{31}$$

where $\pi^0 = l - g$ since clearly the battery charging/discharging capacity $\beta_k = 0$ for all $k \in \{1,\ldots,s\}$.

IV. Savings for a single customer

In this section we define the energy saving for the household PV system depicted in Figure 1. The results in this section allow a single customer to assess the cost-effectiveness of installing a battery of a given size against a *break-even cost*.

A. Energy savings

To examine the effectiveness of QP energy-shifting for a given size battery, we define the *energy savings* (in \$/day). The energy savings are denoted by $\Psi^{\overline{C}}(\mathbf{H})$ and defined by

$$\Psi^{\overline{C}}(\mathbf{H}) := \Sigma^0 - \Sigma^{\overline{C}}(\mathbf{H}). \tag{32}$$

We recall from Section III.C, the energy bill $\Sigma^{\overline{C}}(\mathbf{H})$ is defined for a particular financial policy and selection of weights in $\mathbf{H}$, given load and generation profiles l and g, a battery of a given size $\overline{C}$, with constraints (1)–(3) known and specified. When $\overline{C} = 0$, Σ^0 denotes the base-line energy bill.

For a *break-even cost* denoted by $\zeta(\overline{C})$ (in \$/day), indicating the cost associated with installing battery storage (including the cost of a given size battery $\overline{C}$), we define the *net savings* that accrue to a customer with battery storage. The net savings are denoted by $\theta^{\overline{C}}(\mathbf{H})$ (in \$/day), and defined by

$$\theta^{\overline{C}}(\mathbf{H}) := \zeta(\overline{C}) - \Psi^{\overline{C}}(\mathbf{H}), \tag{33}$$

where $\theta^{\overline{C}}(\mathbf{H}) > 0$ implies a cost-effective installation, $\theta^{\overline{C}}(\mathbf{H}) = 0$ implies cost-neutrality, and $\theta^{\overline{C}}(\mathbf{H}) < 0$ implies no financial benefit for battery storage. Given the net savings, we define the *annual savings* in \$/yr as follows:

$$\Theta^{\overline{C}}(\mathbf{H}) := 365 \times \theta^{\overline{C}}(\mathbf{H}). \tag{34}$$

In the definition of $\Theta^{\overline{C}}(\mathbf{H})$ we assume the initial state of charge χ_0 is the same at the start of each time window T, where T is 24 hours.

B. Special case: zero energy savings

Consider the special case where there is a fixed price for electricity (in \$/kWh) at all installed meters in $\mathscr{M}$, irrespective of power flow direction and time of day. Lemma 2 below demonstrates that under these circumstances, there is no financial incentive for a resident to install battery storage. That is, since the battery acts as an energy time-shifter, the lack of differential pricing at any point in time gives no incentive to energy time-shift.

Lemma 2: Let the cost associated with the initial and final states of charge of the battery, and electricity billing and compensation profiles in the financial policy, be defined as follows:

$$\eta^b = \eta^c \qquad \text{for all } M \in \mathscr{M}, \tag{35}$$
$$\eta^b_j = \eta^b_k = \eta^b_0 = \eta^c_0 \qquad \text{for all } j,k \in \{1,\ldots,s\}, \tag{36}$$

and $\eta^c_j = \eta^c_k$ for all $j,k \in \{1,\ldots,s\}$.

Assume all meters $M \in \mathscr{M}$ are installed such that all power flows are measured and recorded. Then for all choices of battery capacity $\overline{C}$ and weighting matrix $\mathbf{H}$, the energy savings are $\Sigma^{\overline{C}}(\mathbf{H}) = 0$.

Proof: Consider metering topology 2, so that $\mathscr{M} = M_3$. Let

$$\chi_s - \chi^0 = -\sum_{k=1}^{s} \beta_k \Delta, \tag{37}$$

as in equation (2). Substitution of (4), (37) and (35)–(36) into (28) and (30) yields:

$$\begin{aligned}
\Sigma^{\overline{C}}(\mathbf{H}_2) &= T\pi^{\overline{C}}(\mathbf{H}_2)^T\sigma(M_3) + \eta^b_0(\chi_0 - \chi_s)\\
&= T(l-g-\beta)^T\sigma(M_3) - \eta^b_0(\chi_s - \chi^0)\\
&= T(l-g-\beta)^T\sigma(M_3) + \beta^T\sigma(M_3))\\
&= T(l-g)^T\sigma(M_3)\\
&= \Sigma^0,
\end{aligned}$$

where the base-line energy bill Σ^0 is defined in equation (31). The energy savings follow from equation (32):

$$\Psi^{\overline{C}}(\mathbf{H}_2) = \Sigma^0 - \Sigma^{\overline{C}}(\mathbf{H}_2) = 0, \tag{38}$$

and similarly for other metering topologies, provided the meters in $\mathscr{M}$ measure and record all power flows. ■

V. Assessing the benefits of residential battery storage

In this section we assess the financial benefits of QP energy-shifting for a single customer with battery storage in the residential setting shown in Fig. 1. Our approach has two strands. In Section V.A we assume that the battery parameters (1)–(3) are known and fixed, and we seek to determine a preferred way of using the battery by selecting an appropriate weighting matrix $\mathbf{H}$ in the QP. In Section V.B we present a methodology whereby the most cost-effective battery can be selected from a set of candidates.

A. Heuristic approach for selecting H

In Section II, the minimization of expression (5) was presented as a constrained quadratic program (Lemma 1), where the weights h_k in $\mathbf{H}$ were selectable. In this section we consider the specification of the matrix $\mathbf{H}$ with a view to maximizing the annual savings which accrue to a customer with rooftop solar generation, a battery, and residential load as shown in Figure 1.

In practice, the matrix $\mathbf{H}$ that maximizes the annual savings is difficult to obtain, as it depends on a variety of factors including financial policies, metering topologies, and daily variations in load and generation profiles. To address this problem we propose a greedy-search heuristic for obtaining a so-called *preferred* $\mathbf{H}$, which is in turn based upon a *base-line* $\mathbf{H}$ (denoted $\mathbf{H}_0$). When selecting the weights in the preferred $\mathbf{H}$, our rationale is to increase weights when electricity billing is high, and decrease weights when electricity billing is low. We also include constraints to mitigate against numerical difficulties with the solution of the quadratic program in Lemma 1. To this end, weights in the base-line $\mathbf{H}$ are scaled by the minimum cost, and capped at a user-specified maximum.

The basic idea of the heuristic is to increase each weight h_k in $\mathbf{H}_0$ for as long as this increase leads to an increased energy saving in (32), or until a user-defined maximum is reached. To cap the weights h_k we introduce the following saturation operation:

$$\mathrm{sat}_1^{\overline{h}}(h_k) := \begin{cases} h_k, & \text{if } 1 \le h_k \le \overline{h} \\ 1, & \text{if } h_k < 1 \\ \overline{h}, & \text{if } h_k > \overline{h}, \end{cases} \tag{39}$$

where the lower bound is 1 in accordance with the definition of h_k in Section II.A, and $\overline{h}$ is set by the user. The constant $\overline{h}$ is chosen to mitigate against numerical difficulties in solving the QP in Lemma 1. In this paper, we set $\overline{h} = 1000$.

To formalize the definition of the weighting matrix, let

$$\tilde{\eta}_k := \sum_{M\in\mathscr{M}} \eta^b_k(M), \qquad \forall\ k \in \{1,\ldots,s\} \tag{40}$$
$$\eta^\star := \min_{k\in\{1,\ldots,s\}} \tilde{\eta}_k, \tag{41}$$

and define the weighting matrix $\mathbf{H}_0$ as follows:

$$\mathbf{H}_0 := \mathrm{diag}\left[\mathbf{H}_0^{(1)},\ldots,\mathbf{H}_0^{(k)},\ldots,\mathbf{H}_0^{(s)}\right] \tag{42}$$
$$\mathbf{H}_0^{(k)} := \mathrm{sat}_1^{\overline{h}}\left(\tilde{\eta}_k/\eta^\star\right). \tag{43}$$

Given $\mathbf{H}_0$, the proposed heuristic requires the function for energy savings $\Psi(\cdot)$ defined in (32). Recall the energy savings function $\Psi(\cdot)$ requires the constraints and solution to the QP in Lemma 1, and the energy bill $\Sigma(\cdot)$ pertaining to a given metering topology and financial policy, as defined in Section III. To simplify the notation, we use $\Psi(\cdot)$ rather than $\Psi^{\overline{C}}(\mathbf{H})$ to indicate the battery capacity $\overline{C}$ is fixed.

The main loop in the heuristic (lines 6–18) doubles weights in $\mathbf{H}_0$ progressively, from the largest to the smallest element in element in $\mathbf{H}_0$. The set of live indices $\tilde{s}$ keeps track of the indices in $\mathbf{H}_0$ that are yet to be increased, and $\mathbf{I}^{\tilde{s}}$ denotes an s-by-s matrix in which $\mathbf{I}^{\tilde{s}}_{j,j} := 1$ if $j \in \tilde{s}$ and zero otherwise.

Heuristic: Returns the preferred $\mathbf{H}$ given $\Psi(\cdot)$

Input: l, g, $\overline{C}$, $\overline{B}$, $\underline{B}$, χ_0, π^0, $\overline{h}$, $\mathbf{H}_0$,
$\mathbf{H} = \mathbf{I}$, $\Psi_0 = \Psi(\mathbf{H}_0)$, $\tilde{s} = \{1, \dots, s\}$

```
for k ∈ {1,...,s} do
    if π_k^0 = 0 then
        j = {p ∈ {1,...,s} | π_p^0 = 0}
        s̃ = s̃ \ j
        H_0^(j) = 1
while H_0^(s̃) > 1 do
    k* = argmax_s̃(H_0^(s̃))
    H̄_0 = diag[H_0^(1), ..., 2H_0^(k*), ..., H_0^(s)]
    H̄_0^(k*) = sat_1^h̄(H̄_0^(k*))
    Ψ̄_0 = Ψ(H̄_0)
    while Ψ̄_0 > Ψ_0 and H̄_0^(k*) < h̄ do
        Ψ_0 = Ψ(H̄_0)
        H_0 = H̄_0
        H̄_0 = diag[H_0^(1), ..., 2H_0^(k*), ..., H_0^(s)]
        H̄_0^(k*) = sat_1^h̄(H̄_0^(k*))
        Ψ̄_0 = Ψ(H̄_0)
    H = diag[H̄_0^(1), ..., H̄_0^(k*), ..., H̄_0^(s)]
    s̃ = s̃ \ k*
H = H + I^s̃
```

B. Guidance for selecting the battery capacity

In Section IV, we defined the annual savings that accrue to a single customer given a battery of capacity $\overline{C}$. In this section we consider the specification of the most cost-effective battery capacity, given a finite ordered set $\mathscr{C}$ of size m from which to choose, for example $\mathscr{C} = \{0, 1, \dots, 50\}$.

The finite ordered set $\mathscr{C}$ contains m batteries of equivalent charge/discharge limits $\overline{B}$, $\underline{B}$, with initial states of charge $\chi_0 = \rho\overline{C}$, where ρ is known and fixed (e.g $\rho = 0.5$). For $n \in \{1, 2, \dots, m\}$, we define the ordered set by $\mathscr{C} = \{\overline{C}_1, \dots, \overline{C}_n, \dots \overline{C}_m\}$, with $\overline{C}_1 \leq \dots \leq \overline{C}_n \leq \dots \leq \overline{C}_m$.

In formalizing the notion of a *cost-effective battery capacity*, we first define the *maximum battery capacity*, denoted by $\tilde{C}$:

$$\tilde{C} := \min_{\overline{C}_n \in \mathscr{C}} \ \overline{C}_n \qquad \text{s.t } \ \pi^{\overline{C}_n}(\mathbf{H}) = \pi^{\overline{C}_{n+1}}(\mathbf{H}). \tag{44}$$

In the event the specified maximum in (44) does not exist, let $\tilde{C} = \overline{C}_m$. We then eliminate battery capacities greater than $\tilde{C}$ from $\mathscr{C}$ and denoted the reduced set by $\hat{\mathscr{C}}$, as follows:

$$j := \{p \in \{1, \dots, m\} | \overline{C}_p > \tilde{C}\}, \tag{45}$$
$$n := n \setminus j, \tag{46}$$
$$\hat{\mathscr{C}} := \{\overline{C}_1, \dots, \overline{C}_n, \dots, \tilde{C}\}. \tag{47}$$

Given the break-even costs for all $\overline{C}_n \in \hat{\mathscr{C}}$, denoted by $\{\zeta(\overline{C}_1), \dots, \zeta(\tilde{C})\}$, we define the *cost-effective battery capacity* denoted by $\hat{C}$ as follows:

$$\hat{C} := \arg\max_{\overline{C}_n \in \hat{\mathscr{C}}} \theta^{\overline{C}_n}(\mathbf{H}). \tag{48}$$

Recall the net savings $\theta^{\overline{C}_n}(\mathbf{H})$ is defined in (33). In the event $\theta^{\overline{C}_n}(\mathbf{H}) \leq 0$, we say there are no cost-effective battery capacities. In the event (48) has multiple cost-effective battery capacities, let $\hat{C} = \min_{\hat{C} \in \hat{\mathscr{C}}} \ \hat{C}$.

C. Example

We measured load and generation profiles on a winters day in 2011, for each of eight randomly selected low voltage customers, located in Ausgrid's distribution network, NSW Australia. The load and generation profiles l and g for each customer (not shown for lack of space) are defined with $T = 24$ hours, $\Delta = 30$ minutes, and $s = T/\Delta = 48$.

We calculate annual savings for each of the eight customers given financial policies associated with metering topologies 1 and 2 (in Section III.A), via a heuristic method of selecting $\mathbf{H}$ (Section V.A). For each customer the annual savings are calculated for both a 10 kWh or 30 kWh battery. In all cases, the battery constraints are defined in (1)–(3), with $\underline{C} = 0$, $\chi_0 = 0.5\,\overline{C}$, and $\overline{B} = -\underline{B} = 1\,\text{kW}$.[2] For metering topology 1, the length-s billing and compensation profiles (each given in \$/kWh) are $\eta^b(M_1) = \eta^c(M_2) = [0, \dots, 0]^T$, $\eta^c(M_1) = [0.4, \dots, 0.4]^T$, and $\eta^b(M_2) = [\dots, \eta^b_k, \dots]^T$ where $\eta^b_{1-14} = 0.03$, $\eta^b_{15-28} = 0.06$, $\eta^b_{29-40} = 0.3$, $\eta^b_{41-44} = 0.06$, and $\eta^b_{45-48} = 0.03$. For metering topology 2, the length-s compensation and billing profiles (in \$/kWh) are $\eta^c(M_3) = [0.4, \dots, 0.4]^T$, and $\eta^b(M_3) = [\dots, \eta^b_k, \dots]^T$, such that $\eta^b(M_3) = \eta^b(M_2)$. For each customer $\eta^b_0 = \eta^c_0 = 0.03$, and the break-even cost is $\zeta(\overline{C}) = \$0$, irrespective of the battery capacity $\overline{C}$. [3]

Table 1 presents the annual savings for each of the eight customers given a 10kWh or 30kWh battery, for both financial policies relating to metering topologies 1 and 2, and for a baseline and preferred $\mathbf{H}$ (Section V.C). The preferred $\mathbf{H}$ is denoted by $\mathbf{H}_1$ for metering topology 1, and by $\mathbf{H}_2$ for metering topology 2 (Section III.C). We highlight the maximum annual savings per year for the eight customers with bold font, and in this example the financial policy relating to metering topology 1 is preferred for all customers, noting that either financial policy benefits customers 2 and 8 equally. We recall $\hat{C}$ denotes

[2]Often 10kWh and 30kWh batteries allow a faster charge/discharge rate than our chosen charge/discharge rate of $\overline{B} = -\underline{B} = 1\,\text{kW}$. We may justify choosing this charge/discharge rate in situations such as limitations in the cable ratings at the customers premise.

[3]We may justify choosing a break-even cost $\zeta(\overline{C}) = \$0$ for situations where we have a decommissioned electric vehicle (EV) battery, or where we may utilize some portion of an existing EV battery.

TABLE I
ANNUAL SAVINGS FOR CUSTOMERS 1–8

	Battery	Metering Topology 1		Metering Topology 2	
Customer	$\overline{C}$ (kWh)	$\Theta^C(\mathbf{H}_0)$ \$/yr	$\Theta^C(\mathbf{H}_1)$ \$/yr	$\Theta^C(\mathbf{H}_0)$ \$/yr	$\Theta^C(\mathbf{H}_2)$ \$/yr
1	**10**	354	**455**	−119	−19
	30	429	**455**	−44	−18
2	10	756	869	755	869
	30	901	**991**	901	**991**
3	10	237	305	204	272
	30	**306**	**306**	273	273
4	10	262	327	195	260
	30	**328**	**328**	261	261
5	10	342	436	339	433
	30	419	**437**	416	434
6	10	576	704	390	518
	30	661	**705**	475	519
7	10	378	435	−71	−13
	30	**436**	**436**	−13	−13
8	10	981	981	981	981
	30	1530	**1722**	1530	**1722**

a cost-effective battery capacity (Section V.B), and for the preferred $\mathbf{H}$, $\hat{C} = 10$kWh for customer 1, and $\hat{C} = 30$kWh for customers 2–8, highlighted in bold font.

From Table 1, customers 1 and 7 have no financial incentive to QP energy-shift given the financial policy associated with metering topology 2. The significance of selecting an appropriate weighting matrix $\mathbf{H}$, rather than a larger battery capacity is observed for customers 3, 4 and 7, with annual savings associated with metering topology 1. Choosing the base-line weighting matrix $\mathbf{H}_0$, instead of $\mathbf{H}_1$, means these customers require a larger battery capacity ($\overline{C} = 30$kWh) for comparable annual savings (the variation in annual savings is just \$1). However, if we are careful in selecting $\mathbf{H}_1$, a smaller battery capacity may suffice ($\overline{C} = 10$kWh). Therefore these customers are more sensitive to the selection of $\mathbf{H}$, than the battery capacity $\overline{C} \in \mathscr{C}$.

VI. Conclusions

This paper has presented a QP-based framework for assessing the benefits for residential battery storage in conjunction with solar PV, which provides guidance on selecting a cost-effective battery capacity. Given day ahead load and generation profiles, our approach uses known battery constraints and financial incentives to energy time-shift, maximizing the energy savings that accrue to a single customer, whilst reducing the network burden associated with peak load and peak PV generation. Future work will consider extensions to this framework, including a more general network topology and uncertainty in day-ahead predictions of load and generation profiles.

References

[1] N. Garimella and N. Nair, "Assessment of battery energy storage systems for small-scale renewable energy integration," in *Proc. 2009 IEEE TENCON Conf. Region 10 Conference*, Singapore, 23–26 Jan. 2009, pp. 1–6.

[2] N. Li, L. Chen, and S. Low, "Optimal demand response based on utility maximization in power networks," in *Proc. 2011 IEEE Power & Energy Society General Meeting*, San Diego, CA, 24–29 July. 2011, pp. 1–8.

[3] J. Agüero and S. Steffel, "Integration challenges of photovoltaic distributed generation on power distribution systems," in *Proc. 2011 IEEE Power & Energy Society General Meeting*, San Diego, CA, 24–29 July. 2011, pp. 1–6.

[4] G. Hug-Glanzmann, "Coordination of intermittent generation with storage, demand control and conventional energy sources," in *Proc. VIII IEEE Int. Symp. Bulk Power System Dynamics and Control (IREP'10)*, Rio de Janeiro, Brazil, 1–6 Aug. 2010, pp. 1–7.

[5] T. Hubert and S. Grijalva, "Realizing smart grid benefits requires energy optimization algorithms at residential level," in *Proc. IEEE PES Conf. on Innovative Smart Grid Technologies*, Hilton Anaheim, CA, 17–19 Jan. 2011, pp. 1–8.

[6] H. Li, Y. Xu, S. Adhikari, D. Rizy, F. Li, and P. Irminger, "Real and reactive power control of a three-phase single-stage PV system and PV voltage stability," in *Proc. 2012 IEEE Power & Energy Society General Meeting*, San Diego, CA, 22–26 July. 2012, pp. 1–8.

[7] A. Meliopoulos, "A decomposition technique to determine optimal coordinating policies of energy storage plants with electric power system," in *Proc. 18th IEEE Conf. on Decision and Control including Symp. on Adaptive Processes*, vol. 18, December. 1979, pp. 876–878.

[8] Y. Ru, J. Kleissl, and S. Martinez, "Storage size determination for grid-connected photovoltaic systems," *IEEE Transactions on Sustainable Energy*, vol. 4, no. 1, pp. 68 –81, Jan. 2013.

[9] A. Nottrott, J. Kleissl, and B. Washom, "Storage dispatch optimization for grid-connected combined photovoltaic-battery storage systems," in *Proc. 2012 IEEE Power & Energy Society General Meeting*, San Diego, CA, 22–26 July. 2012, pp. 1–7.

[10] T.-Y. Lee, "Operating schedule of battery energy storage system in a time-of-use rate industrial user with wind turbine generators: A multipass iteration particle swarm optimization approach," *IEEE Trans. Energy Convers.*, vol. 22, no. 3, pp. 774–782, September 2007.

[11] J. Xiao, J. Y. Chung, J. Li, R. Boutaba, and J.-K. Hong, "Near optimal demand-side energy management under real-time demand-response pricing," in *Proc. IEEE Int. Conf. on Network and Service Management (CNSM)*, Niagara Falls, ON, 25–29 Oct. 2010, pp. 527–532.

[12] N. L. C. Steele and D. T. Allen, "An abridged life-cycle assessment of electric vehicle batteries," *Environmental Science Technology*, vol. 32, no. 1, pp. 40A–46A, 1998.

[13] S. Brown, D. Pyke, and P. Steenhof, "Electric vehicles: The role and importance of standards in an emerging market," *Energy Policy*, vol. 38, no. 7, pp. 3797 – 3806, 2010.

[14] N. Nair, R. Nayagam, and R. Francis, "New Zealand utility experiences with demand side management," in *Proc. 2008 IEEE Power & Energy Society General Meeting*, Pittsburgh, PA, 20–24 July. 2008, pp. 1–5.

[15] K. Sedghisigarchi, "Residential solar systems: Technology, net-metering, and financial payback," in *Proc. 9th IEEE Int. Conf. on Electrical Power Energy Conference (EPEC09)*, Montreal, Canada, 22–23 Sept. 2009, pp. 1–6.

[16] T. Del Carpio-Huayllas, D. S. Ramos, and R. Vasquez-Arnez, "Feed-in and net metering tariffs: An assessment for their application on microgrid systems," in *Proc. 6th IEEE/PES Int. Conf. on Transmission and Distribution (PTD12)*, Montevideo, Uruguay, 3–5 Oct. 2012, pp. 1–6.