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A General Framework for Decentralized Trading in Electricity Markets

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Abstract

Nowadays the necessary coordination of trades to clear the energy market, while maintaining network security and adequacy of supply, is carried out centrally by a system operator. This paper examines the idea of an autonomously operating power system where energy trading is conducted in a decentralized manner among market players. A general decentralized solution framework is proposed, the main issues and challenges are discussed, and indicative results are presented to illustrate how the overall concept could work.

Introduction

Clearing the energy market involves collecting and managing a large amount of data in order to eventually solve an optimal power flow (OPF) problem. This has always been carried out effectively by a system operator (SO). However, with the advent of the smart grid, the increased penetration of distributed generation, and the need for active participation of consumers in energy management, the need to handle an increasing amount of input data as the system grows and more players enter the market pushes towards a decentralized solution. Resilience (i.e. ability to work even with loss of some of its components), limited information sharing (especially of financial nature), and fast solution speed (i.e. to be fast enough to clear a market in reasonable time) are three important desirable properties of this decentralized scheme. Achieving a decentralized solution involves defining the relevant optimization problem along with its inputs, solving it in a distributed way, establishing the required communication infrastructure and potentially selecting a suitable set of market rules.

A variety of papers have previously dealt with distributed and / or decentralized approaches to power system control and operation. In [1] a Lagrangian Relaxation (LR) is used for the distributed optimization in a three area system. Inherent drawbacks of this technique [2] have prompted the use augmented Lagrangian approaches. A popular approach is based on the so called auxiliary problem principle (APP) [3, 4, 5]. In [6] a proximal point based method (PMM) is used, while [7] introduces a serial implementation of the alternating direction method of multipliers (ADMM) and compares it with the previous two methods. The essential difference between the methods lies in the terms involved in the subproblem objective functions; the results in [7] however do not indicate any significant change in performance. Finally [8, 9, 10] utilize an approach based on the KKT optimality conditions decomposition (OCD). Even though [11] indicates that this approach might reach the optimal solution faster, its convergence condition might not be always easy to verify, nor carrying out the subsequently required conditioning of the problem in a decentralized manner. Overall the above approaches have been tested in systems split in a limited number of areas and their performance has not been evaluated for large degrees of decomposition. Reference [12] uses a parallel implementation of the ADMM method and applies it to randomly generated large scale demand management problems, but does not take Kirchhoff's laws into account. It does however seem to indicate potentially good scalability properties. In [13] a decentralized market clearing approach based on Lagrangian Relaxation is proposed. Details on convergence properties are not provided however, and simulation results were limited to dc power flow equations in a 16 bus system.

This paper proposes a general framework for decentralized trading in electricity markets and presents an implementation of this scheme based on a parallel ADMM method. The latter is adapted to take into account the full AC power flow equations, and its convergence properties are illustrated through simulations on various IEEE test systems of up to 300 buses. In addition, the feasibility of large degrees of OPF problem decomposition, down to the individual bus level, is investigated.

Decentralized System Structure

The underlying idea behind this work is that each individual system component (transmission asset, consumer, generator etc.) or any collection of them may

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be represented by a single agent. In this context an agent is an entity that handles all necessary exchange of information and solves the relevant optimization subproblems, so that it maximizes the economic benefits for those whom it represents. Placement and organization of the agents, as well as the potential benefits from the distributed solution, may differ depending on whether one focuses on transmission or distribution.

In transmission, at the highest degree of decentralization, each bus could be represented by an agent. It may also be argued however that due to the limited number of buses in transmission, decentralization down to individual bus level might not be a solution of interest. The benefits of decentralization in this case might have to do mostly with the fact that, security and adequacy of supply considerations and certain power flow peculiarities may be more easily handled and incorporated into the optimization subproblems of suitably selected areas. On the other hand distribution is where the bulk of energy transactions in a decentralized market may be expected to take place. In contrast with transmission however, system nodes are often not easily accessible and assuming the existence of an agent at each system node could in practice presuppose significant costs. For low voltage networks, the most probable scenario is that a number of aggregators will be handling customers connected in different feeders and coordinate with the agent responsible for managing the medium voltage network and transmitting required price signals.

Electricity markets generally operate in two time frames [14, 15]: the forward market, which operates up to a few hours before the actual power delivery, and the balancing market, which operates in real time. Trading in forward markets to a large extent is carried out through bilateral agreements. These often lack a transparent market price and as a result are slow to adapt to the actual system conditions [16]. The decentralized approach may serve as a mechanism for the discovery of the actual energy price in forward markets by performing a system wide optimization, while taking into account all relevant network constraints, but without requiring information of financial nature to be disclosed (as a centralized approach would do). At the same time it may enable effective demand response in both forward and balancing markets. It should be noted however that the price discovery offered by the decentralized approach comes with a disadvantage. At each moment, even if the market transactions have not been settled yet (i.e. the decentralized scheme has not fully converged) the marginal client has knowledge of this fact. This might allow for some additional gaming possibilities and consequently having a competitive market and adequate demand response capability is critical.

The proposed decentralized structure may be seen on Fig. 1. This work differentiates between two types of agents, namely system and client agents. The former deal with parts of the transmission or distribution system and do not directly include generation or demand. The latter represent either individual consumers or generators, or corresponding aggregating utilities. The proposed distributed optimization algorithm presupposes communication between physically connected agents.

Mathematical Background

The target in clearing the market is to maximize social benefit, in other words maximize the total market players' surplus over the examined time period, i.e. minimize:

$$f_o = \sum_{t\in N_t} \left(\sum_{i \in N_c} u_i(P_i^t, Q_i^t) \right) \tag{1}$$

Where:

- u_i The cost function of the i-th market player.
- N_c A set of n_c elements representing system users / market players.
- N_t A set of n_t time steps covering the optimization period.

The equations following in this subsection describe a specific time step but for simplicity the corresponding notation will be dropped. The basic power flow constraints may be written as:

$$S_i = V_i \sum \left(Y_{ij} V_j\right)^*, i, j \in N_b \tag{2}$$

$$\underline{V_i} \le |V_i| \le \overline{V_i}, i \in N_b \tag{3}$$

$$\left|V_i y_k \left(V_i - V_j\right)\right| \le \overline{T_k}, k \in N_l^{i\to j} \tag{4}$$

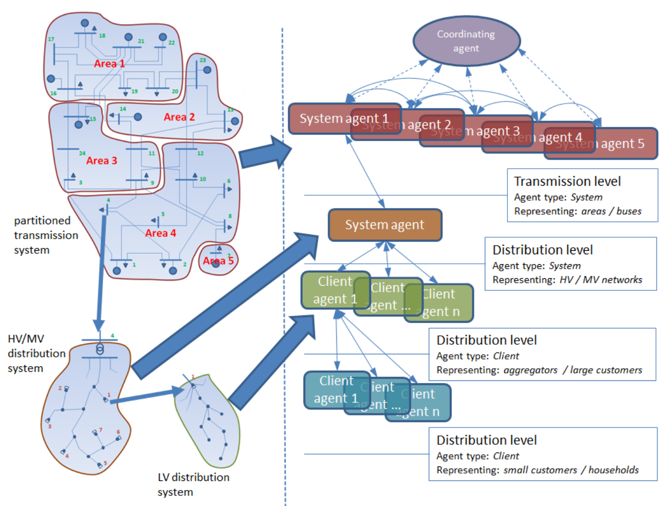

Fig. 1. Example of a power system decomposed into several subsystems, and the corresponding representation with agents. The lines on the schematic on the right indicate required bidirectional communication links.

$$\left|V_j y_k\left(V_i - V_j\right)\right| \le \overline{T_k}, k \in N_l^{i\to j} \quad (5)$$

Where:

- N_b — A set of n_b elements representing system buses.
- N_l — A set of n_l elements representing system lines. The index $i \to j$ indicates that i is the start node of the line and j the end node.
- Y_{ij} — Corresponding element of the bus admittance matrix.
- S_i — Apparent power at i-th bus.
- T_i — Apparent power limit of i-th line.
- V_i — Voltage at the i-th bus.
- y_i — Admittance of the i-th line.

Let $C_h = \{\boldsymbol{S}, \boldsymbol{V}: h_h(\boldsymbol{S}, \boldsymbol{V}) \le 0\}$ denote the corresponding feasible set. This set may be extended to take into account the effects of various contingencies, by replicating the constraints h_h, adding a set of constraints indicating allowable deviations around the normal operating point, and including an expected cost component in the objective function. While the constraints h_h may very well be used to describe a distribution network, it is possible by taking into account the radial nature of the latter, to get to a simpler set of equations, which allows computationally more efficient solutions. One such example is the branch flow model initially proposed in [17], which is a relaxed problem with respect to voltage angle. The corresponding feasible set is denoted as $C_m = \{\boldsymbol{S}, |\boldsymbol{V}|: h_m(\boldsymbol{S}, |\boldsymbol{V}|) \le 0\}$.

Down at the network user level the constraints vary depending on the user type. At the very least these consist of active and reactive power limits and some kind of relation between them. Commonly time-linkage constraints are also involved, which define relationships between power outputs of consecutive hours. Again let $C_l = \{\boldsymbol{S}: h_l(\boldsymbol{S}) \le 0\}$ denote the feasible set at this level. The optimal solution of the overall power flow problem should lie in the set $C = C_h \cap C_m \cap C_l$. This set is non-linear and non-convex. Even though the objective function (1) is separable, due to the voltage and power vectors the constraints are not.

Decomposition approach

As a first step to bring the problem into a suitable form for decomposition, one fictitious bus is introduced at the middle of each line that connects systems which are managed by separate agents. In a similar fashion a fictitious node may be introduced for each demand or generation block. The variables associated with that particular bus are duplicated as seen on Fig. 2. Let $\boldsymbol{U}_e = [\boldsymbol{S}_e; \boldsymbol{V}_e]$ be the duplicated variables vector, and $\boldsymbol{U} = [\boldsymbol{S}; \boldsymbol{V}; \boldsymbol{U}_e]$ a vector of all control variables. Grouping up the corresponding coupling equations yields a set of linear constraints $C_c = \{\boldsymbol{U}_e: h_c(\boldsymbol{U}_e) = 0\}$. Furthermore the set C has to be suitably modified to take into account the additional variables, yielding the new extended set $C_e = \{\boldsymbol{U}: h_e(\boldsymbol{U}) \le 0\}$. The solution of the OPF problem should belong in the set $C_e \cap C_c$, with C_e being a decomposable set with respect to initial system buses and individual demand or generation blocks.

The constraints h_c may now be handled using any decomposition method [18]. One such possibility is using an augmented Lagrangian approach and achieving the decomposition using the ADMM method [19] which requires setting a single parameter and yields a rather simple multiplier update step. First the initial OPF problem is reformulated as follows:

$$\min_{z, U \in C_e} \{f_p(\boldsymbol{U}) + g(\boldsymbol{z}): \boldsymbol{z} = \boldsymbol{U}_e\} \quad (6)$$

Where g is an indicator function into the set $C_c' = \{\boldsymbol{z}: h_c(\boldsymbol{z}) = 0\}$. The augmented Lagrangian is:

$$\mathcal{L}_\rho(\boldsymbol{U}, \boldsymbol{z}, \boldsymbol{\lambda}_e) = f_0(\boldsymbol{U}) + g(\boldsymbol{z}) + \boldsymbol{\lambda}_e^T(\boldsymbol{U}_e - \boldsymbol{z}) + (\rho/2)\|\boldsymbol{U}_e - \boldsymbol{z}\|_2^2 \quad (7)$$

Then starting from an estimate of $\boldsymbol{\lambda}_e$ and $\boldsymbol{z}$ the following steps are repeated until convergence:

$$\boldsymbol{U}^{k+1} = \operatorname{argmin}_{\boldsymbol{U}} \mathcal{L}_\rho(\boldsymbol{U}, \boldsymbol{z}^k, \boldsymbol{\lambda}_e^k) \quad (8)$$

$$\boldsymbol{z}^{k+1} = \operatorname{argmin}_{z} \mathcal{L}_\rho\left(\boldsymbol{U}^{k+1}, \boldsymbol{z}, \boldsymbol{\lambda}_e^k\right) \quad (9)$$

$$\boldsymbol{\lambda}_e^{k+1} = \boldsymbol{\lambda}_e^k + \rho(\boldsymbol{U}_e^{k+1} - \boldsymbol{z}^{k+1}) \quad (10)$$

These equations may be simplified into the following:

$$\boldsymbol{U}^{k+1} = \operatorname{argmin}_{U \in C_e}\{f_p(\boldsymbol{U}) + (\boldsymbol{\lambda}_e^k)^T\boldsymbol{U}_e + (\rho/2)\|\boldsymbol{U}_e - \boldsymbol{z}^k\|_2^2\} \quad (11)$$

$$\boldsymbol{z}^{k+1} = \Pi_C\{\boldsymbol{U}_e^{k+1} - \boldsymbol{z}\} \quad (12)$$

$$\boldsymbol{\lambda}_e^{k+1} = \boldsymbol{\lambda}_e^k + \rho(\boldsymbol{U}_e^{k+1} - \boldsymbol{z}^{k+1}) \quad (13)$$

Where Π_C is the Euclidian projection on the set C_c. The optimization problem of (11) is decomposable. Equation (12) presupposes that the initial estimate for the Lagrange multiplier of a fictitious bus or node is selected so that $(\boldsymbol{\lambda}_e^k)^T\boldsymbol{z} = 0$. Equations (11) to (13) are solved locally by each agent but require exchange of information on local power and voltage estimates between physically connected agents. The only actually exchanged information of financial nature is the estimate of marginal price (i.e. Lagrange multiplier) at the particular node, which might be considered as an advantage of the method.

A suitable communications network would be required to support this scheme and would affect its speed and

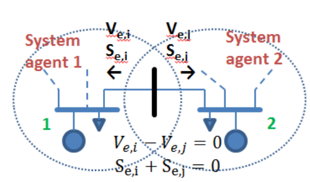

Fig. 2. Fictitious buses and duplication of variables.

reliability. The time to convergence is equal to the sum of maximum latency and computation time among all agents per iteration. Currently there is a large variety of communication technologies available but no fully fledged standard for smart grids. A review of various types of communication networks may be found in [20] and some indicative latency numbers in [21].

Necessary and sufficient conditions for convergence are primal and dual feasibility [19]. These may be expected to be satisfied if $\max \rho \|r^k\|_2^2 \leq \epsilon$, which is the criterion used in this work. It is adequate that convergence be checked only by transmission level agents. Convergence information may either be sent by each system agent to the central coordinating agent or it may be propagated throughout the network at the cost of additional iterations.

Convexity & Problem Formulation

A major issue with the OPF problem is the fact that the involved equations are non-convex. An approach in convexifying the OPF problem based on semidefinite programming (SDP) was presented in [22]. However as [23] indicates this relaxation is not always tight. For radial networks convexification, either in the form of an SDP formulation [24] or a conic relaxation approach [25], is tenable with restrictions regarding the objective function. For meshed networks, especially when considering a variety of additional constraints, convex approximations may not be always possible. Still this does not mean that a meaningful solution may not be reached, as often the duality gap is small, and any issues regarding the non-convex nature of certain system components might be easier to handle within the smaller subproblems.

Indicative Results

The optimization problems solved in this section involve a single time period where generating units have already been committed. The presented results however, which illustrate the method's convergence properties for a variety of cases, have implications for the application of the decentralized scheme both in balancing and forward markets. The simulations that follow involve a fixed penalty factor (set at the same value for all simulations) For all test cases the results were compared with centralized OPF solutions and were found to be within acceptable accuracy. Tests were carried out with both the full AC formulation and a non-linear DC approximation by neglecting the reactive power equations and setting voltages equal to unity.

A Base Working Case

The example presented here is based on the IEEE RTS 24 bus system. Load blocks and generators are randomly split into several smaller blocks. These might be considered to be aggregators representing parts of local demand and generation. Each bus is represented by an agent. At iteration 0 the coordinating agent sends a signal to all system agents indicating the start of trading. After this moment individual client agents may enter the market or adjust their position at will by communicating with their corresponding upper level agents. At iteration 500 the market closes. After this point no new agents are allowed to enter the market and participate in the optimization process. As may be seen in Fig. 3, in the meantime the distributed optimization process works continuously. When convergence is detected each system agent sends relevant information to the coordinating agent. In this example the time (i.e. iteration) in which a client enters the market was randomly drawn from a uniform distribution. Some iterations after the market closure, the optimization gradually converges. The number of iterations (time) to market closure was arbitrarily selected and is not necessarily indicative of a real situation. In forward markets a long time period (corresponding to a high number of iterations) may be allowed before the market closes, in real time markets this time will be more limited.

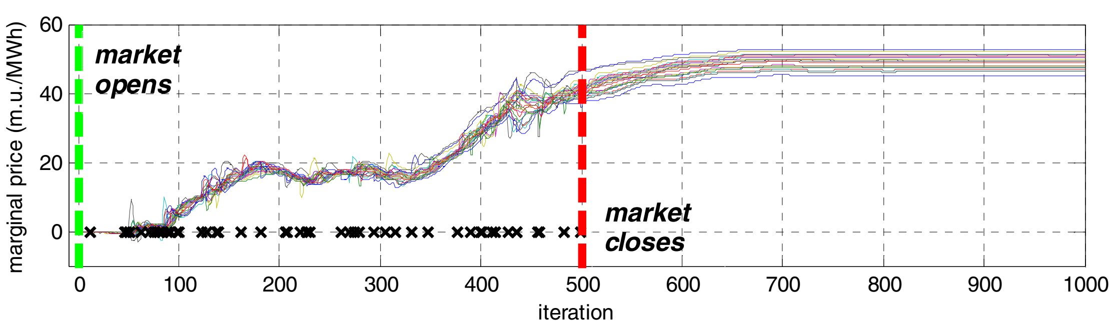

Fig. 3. Convergence process for buses marginal prices. Marked points on the horizontal axis indicate iterations where new clients enter the market.

Convergence Considerations

The first set of tests here involves decomposition down to individual bus levels with IEEE test systems (24, 57, 118 and 300 buses). All client agents were assumed to be connected from the first iteration. The results may be seen in Fig. 4. Overall, system size (i.e. the number of cooperating system agents) is an important parameter. However the decrease in iterations for the systems of 57 and 118 buses indicate a high dependence of the convergence process on the characteristics of the system and of the particular case to be solved. It may also be seen that DC and AC OPF have slightly different scaling performance. A total of 445 iterations were required for the convergence of non-linear DC in the IEEE RTS with decomposition to individual bus level. This is approximately 8% more than the required number of iterations in the full AC case. The tolerance was set to $\epsilon = 10^{-3}$. In a second set of tests the IEEE RTS was split into a varying number of areas, each containing a random number of buses and managed by a single agent. The effect of increased system decomposition on the convergence of the algorithm may be seen on Fig. 5.

Convergence issues were further investigated through tests on the IEEE RTS system. Tests with different demand levels and the same generating unit configuration indicated a varying number of iterations to convergence of approximately up to 3 times the iterations required for the base scenario. Another set of tests involved randomly selected cases with outages of generators or transmission lines or their operation at reduced capacity. For cases where load curtailment was required it was assumed that the corresponding cost is about 10 times the marginal cost of the peak load case. In general the number of required iterations for all cases without significant marginal prices deviations was near that of the base case. In cases where, due to system peculiarities, there were large price deviations, or load curtailments were required convergence was slow ($> 3 \cdot 10^3$ iterations). Further tests however indicated that convergence in these cases was significantly improved ($< 2 \cdot 10^3$ iterations) by an increased penalty factor value at the later stages of the simulation.

Aggregation Considerations

This example investigates how convergence is affected by an increase in the number of participating client agents in the network. Demand blocks in the IEEE RTS system were divided randomly in several smaller ones. The effect on the number of iterations assuming the decomposition is carried out at a single go (i.e. all subproblems solved in parallel) may be seen on Fig. 6. The increased number of iterations is due to the fact that as the algorithm progresses to convergence the primal residuals are significantly reduced. For a client agent managing a very small nominal power this implies small Lagrangian multiplier updates, and as a result a higher number of iterations. The existence of suitably designed aggregators could offer a significant improvement in convergence.

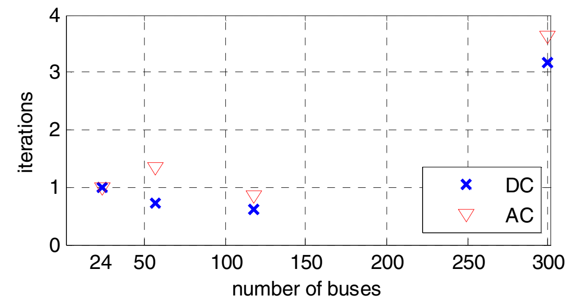

Fig. 4. Effect of system size on the number of iterations to convergence (normalized by the number of iterations required for the base IEEE RTS case).

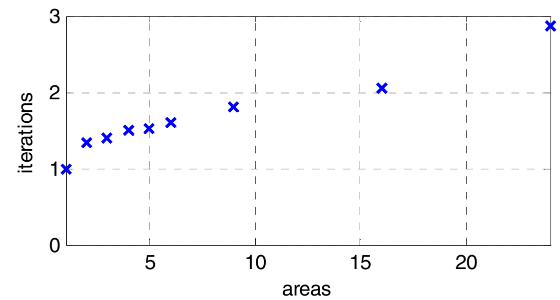

Fig. 5. Iterations to convergence (normalized by the number of iterations required for the single system agent case) as a function of the number of areas / system agents.

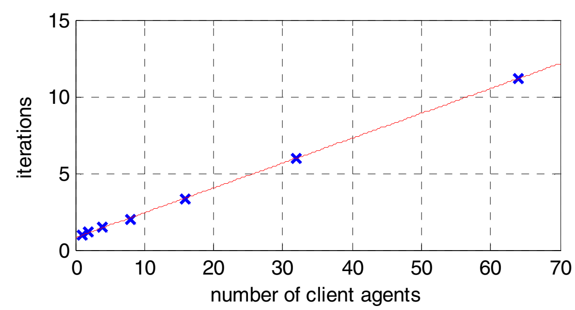

Fig. 6. Effect of an increased number of clients (normalized by the number of clients in the base case) on the number of iterations to convergence (normalized by the number of iterations required for the base case).

Conclusions

A general framework that could enable decentralized trading has been presented and the relevant basic challenges and issues have been outlined. Particular emphasis was placed on the decomposition of the relevant optimization problem on the transmission level. It has been shown through simulations that convergence speed is highly dependent not only on the number of participating agents in the market, but also on the particular case to be solved. The presented decentralized approach can be potentially applicable to suitably partitioned systems. Future work in this area involves investigating the coordination between higher and lower level agents, incorporating ancillary services markets into the optimization process and improving the convergence properties of the method itself. The level of abstraction in the system representation is also a question of particular importance.

References

[1] A. J. Conejo and J. A. Aguado, "Multi Area Coordinated Decentralized DC Optimal Power Flow," *IEEE Transactions on Power Systems,* vol. 13, no. 4, pp. 1272-1278, Nov. 1998.

[2] J. Batut and A. Renaud, "Daily Generation Scheduling Optimization with Transmission Constraints: A New Class of Algorithms," *Trans. Power Systems,* vol. 7, no. 3, pp. 982-989, 1992.

[3] G. Cohen, "Auxiliary Problem Principle and Decomposition of Optimization Problems," *Journal of Optimization Theory and Applications,* vol. 32, no. 3, pp. 277-305, Nov. 1980.

[4] B. H. Kim and R. Baldick, "Coarse-Grained Distributed Optimal Power Flow," *IEEE Trans. on Power Systems,* vol. 12, no. 2, pp. 932-939, May 1997.

[5] J. Contreras, A. Losi, M. Russo and F. F. Wu, "DistOpt: A Software Framework for Modeling and Evaluating Optimization ProblemSolutions in Distributed Environments," *Journal of Parallel and Distributed Computing,* vol. 60, no. 6, pp. 741-763, June 2000.

[6] B. H. Kim, "An Efficient Implementation of Decentralized Optimal Power Flow," *Journal of Electrical Engineering & Technology,* vol. 2, no. 3, pp. 335-341, 2007.

[7] B. H. Kim and R. Baldick, "A Comparison of Distributed Optimal Power Flow Algorithms," *IEEE Trans. Power Systems,* vol. 15, no. 2, pp. 599-604, May 2000.

[8] A. J. Conejo, F. J. Nogales and F. J. Prieto, "A Decomposition Procedure Based on Approximate Newton Directions," *Mathematical Programming,* vol. 93, no. 3, pp. 495-515, Dec. 2002.

[9] A. Bakirtzis and P. N. Biskas, "A Decentralized Solution to the DC OPF of Interconnected Power Systems," *IEEE Trans. on Power Systems,* vol. 18, no. 3, pp. 1007-1013, Aug. 2003.

[10] G. H. Glanzmann and G. Andersson, "Decentralized Optimal Power Flow Control for Overlapping Areas in Power Systems," *IEEE Trans. Power Systems,* vol. 24, no. 1, pp. 327-336, Feb. 2009.

[11] M. Arnold, S. Knopfli and G. Andersson, "Improvement of OPF Decomposition Methods Applied to Multi Area Power Systems," in *IEEE PowerTech*, Lausanne, Switzerland, 2007.

[12] M. Kraning, E. Chu, J. Lavaei and S. Boyd, "Message Passing for Dynamic Network Energy Management," **available online**.

[13] D. Papadaskalopoulos and G. Strbac, "Decentralized Participation of Electric Vehicles in Network Constrained Market Operation," in *Innovative Smart Grid Technologies (ISGT)*, Berlin, Germany, 2012.

[14] PJM, "Manual 11 - Rev. 59: Energy & Ancillary Services Market Operations," 2013.

[15] N. Grid, "National Electricity Transmission System Seven Year Statement," 2011.

[16] S. Stoft, Power System Economics, IEEE Press, 2002.

[17] M. E. Baran and F. F. Wu, "Optimal Capacitor Placement on Radial Distribution Systems," *IEEE Trans. Power Delivery,* vol. 4, no. 1, pp. 725-734, 1989.

[18] A. J. Conejo, E. Castillo, R. Minguez and R. Garcia-Bertrand, Decomposition Techniques in Mathematical Programming: Engineering and Science Applications, Springer, 2006.

[19] S. Boyd, N. Parikh, E. Chu, B. Peleato and J. Eckstein, "Distributed Optimization and Statistical Learning via the Alternating Direction Method of Multipliers," *Foundations and Trends in Machine Learning,* vol. 3, no. 1, pp. 1-122, 2010.

[20] V. C. Gungor and F. C. Lambert, "A Survey on Communication Networks for Electric System Automation," *Computer Networks,* vol. 50, no. 7, pp. 877-897, 2006.

[21] W. Wang, Y. Xu and M. Khanna, "A Survey on the Communication Architectures in Smart Grid," *Computer Networks,* vol. 55, pp. 3604-3629, 2011.

[22] J. Lavaei and S. H. Low, "Zero Duality Gap in Optimal Power Flow Problem," *IEEE Trans. on Power Systems,* vol. 27, no. 1, pp. 92-107, 2012.

[23] B. C. Lesieutre, D. K. Molzahn, A. R. Borden and C. L. DeMarco, "Examining the Limits of the Application fo Semidefinite Programming to Power Flow Problems," in *49th Annual Allerton Conference on Communication, Control & Computing*, Monticello, USA, 2011.

[24] S. Bose, D. F. Gayme, S. Low and K. M. Chandy, "Optimal Power Flow Over Tree Networks," in *49th Annual Allerton Conference on Communication, Control & Computing*, Monticello, USA, 2011.

[25] M. Farivar and S. H. Low, "Branch Flow Model: Relaxations and Convexification - Part I," *IEEE Trans. Power Systems.*