

Application of Gramians method for Smart Grid investigations on the example of the Russky Island Power Network

A. Grobovoy, V. Shipilov, A. Arestova
Power System Emergency Control Lab
(Russia)

I. Yadykin, V. Afanasyev, A. Isakov, D. Kataev
Trapeznikov Institute of Control Sciences
Russian Academy of Sciences
(Russia)

Abstract

The paper deals with a method of stability investigation for Smart Grid on the example of the Russky Island power network at the end of continental Russia in the Pacific region. The new forms of cross-Gramians and Gramians of controllability and observability for both finite and infinite time were used for the investigation. The software package MATLAB/Simulink was applied for the model creation and stability investigation of the Russky Island power network. The advantage of the new method of examination for stability compared with the use of H_2 -norm transfer matrix of the system is shown.

Introduction

Basis for the Island Russky network consideration

One of the most convenient places for realization of the so-called Smart Grids concept in Russia can become the Russian Far East Interconnected Power System. This power grid might contain several mini-grids each of whose might comprise several micro-grids. The ambitious project has been undertaken by the Russian Federal Grid Company (FGC) in order to realize above mentioned possibilities. The electric network of Island Russky at the end of continental Russia in the Pacific region may become by one of the objects for realization of these intentions.

Recently established infrastructure of power network at Russky Island was put into operation last year. The power network contains four power sources, namely Combined Heat and Power (CHP) plants which are interconnected each other with the use of the 35 kV cable lines. The existing network is noted by continuous lines in Fig.1. The connection of the Russky Island power network with continental part of the power grid is fulfilled by two 220 kV transmission lines. The rated voltages of the substation which is the point of connection with the power grid are

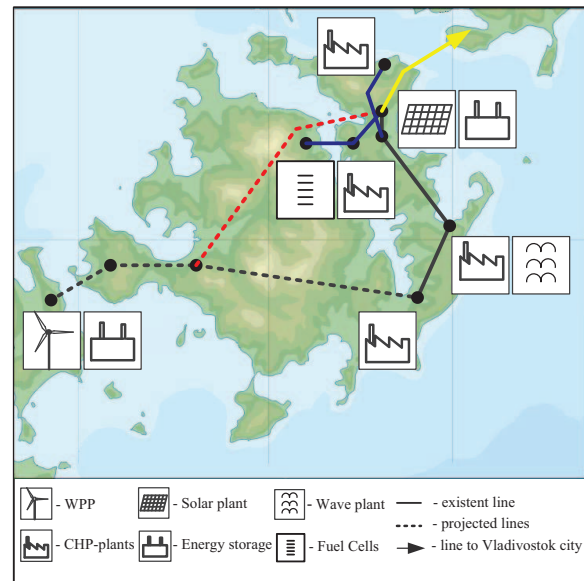


Fig.1 Possible structure of Russky Island VPP

220/110/35 kV. This circumstance gives us the right to assume that the future development of the power network can also be realized with the use of 110 kV transmission lines.

If we assume that the development of distributed generation at Russky Island will be based on the creation of the wind power plant (WPP) at Popov Island which is located hereabout Russky Island (this is planned by some companies), then depending on the WPP rated power some schemes for transmission system can be offered. One of the potential variants of such scheme is also shown in the picture. The red dotted line in Fig. 1 indicates a possible 110 kV transmission line that could enhance the existing and eventual transmission lines of 35 kV which are shown by the black dotted lines.

Taking into account the fact that the Russky Island belongs to the regions which are the most secured by solar

radiation regions, there exist the conditions for the use of the solar energy here. It is a reason why the pictogram of "Solar plant" is noted at the sketch of the power network. Now there is a good perspective for implementation of the so-called hydrogenous energy in Smart Grids. Therefore the icon "Fuel cells" can be represented on the picture too. Because the island is owned by the Pacific Ocean, this allows us to consider the use of the so-called wave power plant for power supply of consumers. This icon is also shown in Fig. 1. The implementation of distributed generation stipulates the necessity to use the variety energy storage devices. It is the mandatory condition for secure functioning of distributed generation. That is why the appropriate pictograms are shown in the picture. Thus, the power network of Russky Island might become in the future by a classical example of Virtual Power Plant (VPP). This can become a very convenient place for investigation of the Smart Grids technologies.

The reason for using Gramians method

The development of the Smart Grid conception is in conditions of constant advancing IT technologies. This gives engineers the ability improving the automated supervising and the automatic control in order to improve the security and reliability of power systems. In these conditions, any added information about the possible directions of the improving of controllability of power system can become helpful for operators and dispatchers to make the right decisions in the process of operation of the power system. Under executing automatic control that may demands of tens milliseconds for forming control actions, it is important to be able to add some time for realization of control algorithms. The Gramians method, in some cases, can allow doing it. This method can give a dispatcher the information about the possible nature of evolution of the emergency. In addition, it can give some additional time for automatic device to implement more sophisticated algorithms to generate the control actions for prevention of instability. The traditional method of finding instability for the so-called Multiple Input Multiple Output (MIMO) systems consists in the determining the moment of transition H₂-norm of dynamics matrix into infinity. One can improve this method, including for the applications of supervising and control in power systems.

The Gramians method is a new method for assessing the degree of static stability of the power system, based on a new mathematical approach to solving both Lyapunov and Sylvester differential and algebraic equations. This approach is based on the decomposition of Gramian matrix, which is the solution of the Lyapunov or Sylvester, on the spectrum of matrices that form these equations.

Theoretical background

Mathematical model of power system is described by the following nonlinear differential-algebraic system of equations

$$\begin{aligned} \dot{x}(t) &= f(x, u, t), \quad x(t_0) = 0, \\ M(x, t)x(t) &= N(x, t)u(t). \end{aligned} \quad (1)$$

Linearized model, relative to a fixed power system mode, is described by a linear differential-algebraic system of equations [1]

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \quad x(t_0) = 0, \\ Mx(t) &= Nu(t). \end{aligned} \quad (2)$$

We assume that the matrix M is nonsingular. Then (2) can be rewritten as

$$\begin{aligned} \dot{x}(t) &= A_1x(t) + Bu(t), \quad x(t_0) = 0, \\ A_1 &= A + M^{-1}N, \end{aligned} \quad (3)$$

Consider the continuous differential and algebraic Lyapunov equation for the system (3) of the following form

$$\begin{aligned} \frac{dP(t)}{dt} &= AP(t) + P(t)A^T + R, \quad P(0) = 0_n, \\ AP(\infty) + P(\infty)A^T + R &= 0. \end{aligned} \quad (4)$$

Let us give a name to the expression

$$P_k^c = \sum_{j=0}^{n-1} \sum_{\eta=0}^{r-1} \frac{s_k^j (-s_k)^\eta}{N'(s_k)N(-s_k)} A_j B B^T A_\eta^T$$

as an infinite sub-Gramian of a single root of the characteristic polynomial of matrix A . Here A_j, A_η are the Faddeev's matrices in the expansion of the resolvent matrix of the dynamics of the system (3) [2]-[3]. In accordance with this the infinite gramian of controllability is defined as

$$P^c = \sum_{k=1}^n P_k^c$$

The following theorem provides a method for solving Lyapunov algebraic equations.

Theorem [4]. Consider a linear completely controllable and observable continuous stable system of the form (1). Assume that the matrix A is Hurwitz and the following condition has been met

$$s_k + s_\lambda \neq 0 \quad \forall k = 1, 2, \dots, n, \quad \forall \lambda = 1, 2, \dots, n.$$

Then the following assertions are valid:

Assertion 1. The sum of these sub-Gramians corresponding to real roots and pairs of complex conjugate roots is the infinite gramian of controllability of the system:

$$P^c = \sum_{k=1}^l P_k^c + \sum_{k=1}^m (P_{k1}^c + P_{k1+1}^c) \quad (5)$$

where the index $k1 + 1$ corresponds to the root, paired with the root $k1$.

Assertion 2. Sub-Gramians are the matrix quadratic forms of the form:

$$P_k^c = \sum_{j=0}^{n-1} \sum_{\eta=0}^{r-1} \frac{s_k^j (-s_k)^\eta}{N'(s_k) N(-s_k)} A_j B B^T A_\eta^T, \quad (6)$$

$$A_{(k)} = \sum_{j=0}^{n-1} \frac{s_k^j A_j}{N'(s_k)} = \lim_{s \rightarrow s_k} \frac{(s - s_k) \sum_{j=0}^n s_k^j A_j}{N(s)} = \text{Res}(Is - A)^{-1} \Big|_{s=s_k};$$

Assertion 3. The following relation exists for pairs of complex conjugate roots

$$P_{k1}^c + P_{k1+1}^c = 2 \text{Re } P_{k1}^c = -2 \text{Re } A_{k1} B B^T (A^T + s_{k1} I)^{-1} \quad (7)$$

Assertion 4. Sub-Gramian corresponding to a particular root of the characteristic equation is the only solution of linear algebraic equations of the form

$$P_k^c (s_k I_k + A^T) = -A_{(k)} B B^T, \quad (8)$$

$$P_k^c = -A_{(k)} B B^T (s_k I_k + A^T)^{-1}. \quad (9)$$

The transfer function of SISO system can be defined as

$$W(s) = \frac{y(s)}{u(s)} = C(Is - A)^{-1} B =$$

$$= \frac{CA_{n-1}Bs^{n-1} + \dots + CA_1Bs + CA_0B}{N(s)} =$$

$$= \frac{b_{n-1}s^{n-1} + \dots + b_1s + b_0}{N(s)}.$$

Then the squared H_2 -norm for the SISO (single-input, single-output) system can be calculated as

$$\|W\|_2^2 = \sum_{k=1}^n G_k,$$

$$G_k = \sum_{j=0}^{n-1} \sum_{\eta=0}^{r-1} \frac{s_k^j (-s_k)^\eta}{N'(s_k) N_m(-s_k)} b_j b_\eta,$$

Method for finding power transfer limit

- Forming of the matrix representation in the state space of the linearized power system model of the form (2) for the basic mode.
- Determining the eigenvalues of the dynamics matrix $(A + Id)$ with the sliding $d > 0$
- Calculating of sub-Gramians for weak modes and their H_2 -norm
- Finding the weak-critical mode, localization of the critical of the network, and finding power transfer limit.

Used algorithm

The quadratic matrix coefficients of the numerator of the transfer function for one mode such as

$$G_k = \sum_{\eta=0}^{n-1} \frac{s_k^j (-s_k)^\eta}{N'(s_k) N_m(-s_k)}, \quad (10)$$

are in general the complex ones.

Since the square of the norm of the transfer function of the system is non-negative, then the matrix of the square of H_2 -norm of the transfer function in (10) is a real symmetric and positive definite. However, accordingly with (10), matrix of the quadratic form of G_k is not symmetric. We define the Hermitian component of G_k in the form

$$H(G_k) = \frac{1}{2} (G_k + G_k^*). \quad (11)$$

Taking into account (11) we have

$$b^T G b = \sum_{k=1}^n b^T H(G_k) b = \text{Re} \sum_{k=1}^n b^T H(G_k) b. \quad (12)$$

The square of H_2 -norm of the quadratic form with matrix G_k defines a measure of energy of a single mode in the transients caused by the application of the pulse signal in the form of the Dirac delta function $\delta(t)$ at the input of the system. The physical meaning of (11) consists in that the measure of the energy is the sum of the energies of all modes of transients. The spectral decomposition of the squared norm of the form (5) gives us possibility to reveal the role of real and complex roots in formatting of the measure for ill-stable dynamical systems, which is particularly important when considering the transient operation of systems near the boundaries of stability.

The matrix of the quadratic form G_k for single mode during transients is a solution of an algebraic equation of Sylvester in the form

$$s_k I G_k + G_k A^T + CA_{(k)} B B^T C^T = 0, \quad (13)$$

$$A_{(k)} = \sum_{j=0}^{n-1} \frac{s_k^j A_j}{N'(s_k)} = \text{Res}(Is - A)^{-1} \Big|_{s=s_k}.$$

$$G_k = -A_{(k)} B B^T [I(-s_k) - A^T]^{-1}, \quad \|W\|_2^2 = \frac{b(s_k)b(-s_k)}{N'(s_k)N(-s_k)}, \quad (14)$$

For pairs of complex conjugate roots the following relation is valid

Fig. 3 Configuration of excitation system and speed control in the terms of the MATLAB/Simulink/SimPowerSystems package

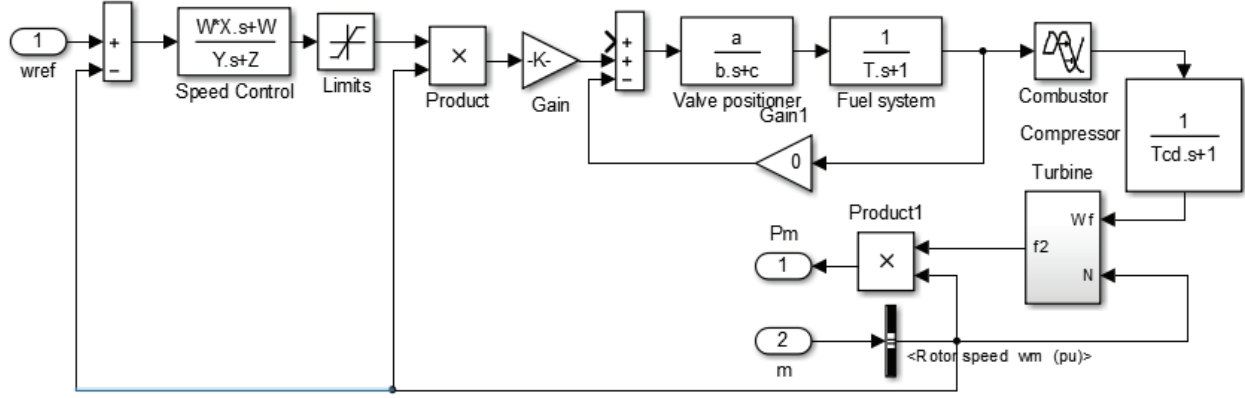


Fig. 4 The segment of Rowen's gas-turbine model used in this study case

Figure 4 shows the modification of the well known Rowen's model [5] for the gas turbine that was used in this investigation. In this study case only one channel of control is used for the simulation. The feature of this approach consists in the neglect of the channels of the temperature control of the exhaust gases and the acceleration loop channel.

Figure 5 illustrates the dynamics of the power network model obtained by simulation of the transients when emergency segregating of the Russky Island power network from the continental grid when transferring pre-fault power flow of 14 MW which arise as result of stopping of two from five generators which are represented by generator G1. One should pay the main attention on the behavior of G4 which is the most affected from the side of the disturbances that confirms indirectly the results obtained in the next paper segment.

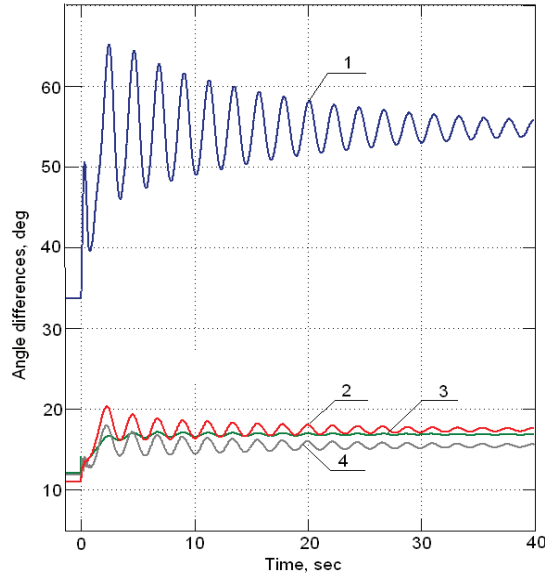


Fig. 5 Angle curves: angle differences between the emf vectors of (1) G1 and G4, (2) G1 and G3, (4) G1 and G2, (3) is own angle of G1

Applying of Gramians method

The algorithm of the Gramians and sub-Gramians calculation has been realized in the MATLAB software package. It was applied to the Russky Island power network model. The experiment was consisted in the step-by-step increasing of the loads by multiplying the basic value on the factor α in the range of from 1 to 4. Linearization of the system, determination of its spectrum, and calculation of the sub-Gramians norms have been made at each step of the regime change. Analysis of obtained modal parameters, resulting from the experiment, gives us possibility to reveal two of the most weakly stabilized modes. This is the internal oscillatory mode, which consists of eight modal components, generated by G4, and also inter-machine oscillatory mode, which includes four modal components generated by G1, G2, G3, and G4.

The curves of these modes in the range of variation of the factor α from 3.3 to 4.0 are shown in Fig.6. The curve of H_2 -norm of the dynamic matrix of the system is also shown for comparison.

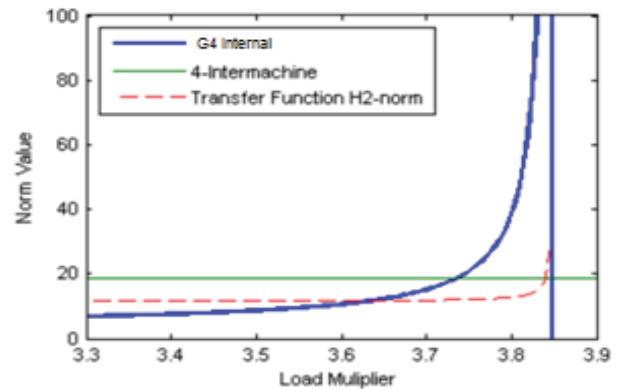


Fig. 6 Collation of Gramians and classical H_2 -norm movement

It can be seen from the results of Gramians method applying, which under reaching a value of α of about 3.85 the system lost its stability. The reason is the excess of the power transfer limit in section G3-G4. It is also evident that the Gramians method allowed discovering the dangerous tendencies in the system much earlier than the analysis of H_2 -norm of the transfer function.

Among other things, it could be useful to use the Gramians method for supervisory control systems in power grids. In addition, this gives us right to recognize the Gramians method as the possible thing for improving the system of automatic prevention of instability in the so-called virtual power plants and microgrids. The Gramians method may become a mandatory attribute of multi-agent systems responsible for reliability of such grids.

Conclusions

The Sub-Gramians method, based on calculating the norms with the use of the Gramian "sliding" value in the static stability problem, allow calculating of the stability system degree (d_{em}) for the power grids on the similarity as it can be made with the use of the modal analysis method [6].

The Sub-Gramians method allows calculating the amount of energies of ill-stable modes in the total energy and takes into account the mutual influence of the modes to the power transfer limit.

Analysis of the behavior of H_2 -norm of sub-Gramians at several ill-stable modes allows defining the beginnings of tendency towards approach to the power transfer limit at the earlier stage of calculating [7].

The virtual power plant that could be created at Russky Island on the basis of the existing power network can become an experimental platform for testing Smart Grid technologies in Russia. The Gramians method is one of the most promising ways that could be studied there.

Appendix

Table 1. Transformer data

Symbol	2*T1	2* T2	2*T4	2*T3	2*T5
S (MVA)	63	25	25	16	6.3
Ratio	220/110/35	35/10	10/6.3	35/10	35/10
Pcu (kW)	215/-/-	120	110	85	46.5
Uk (%)	11/35.7/22	10.5	10	10	7.5

Table 2. Generator data

Symbol	G1	G2	G3	G4
Number	5	2	2	1
Sn , MVA	7.33	2	7.33	1.8
Pn, MW	6.23	1.8	6.23	1.62
Un, kV	10	6.3	10	10
cosfn	0.85	0.9	0.85	0.9
f, Hz	50	50	50	50
Speed, rpm	1500	1500	1500	1500
Reactances in p.u.				
xd (%)	238	238	238	375
xd' (%)	33.6	33.6	33.6	26.7
xd''(%)	24	24	24	14.8
xq (%)	121	121	121	225
xq''(%)	23	23	23	15
xl	18	18	18	20
Time constants				
Td0' (s)	9.67	9.67	9.67	2.6
Td0'' (s)	0.05	0.05	0.05	0.019
Tq' (s)	0.337	0.337	0.337	0.267
Tq'' (s)	0.029	0.029	0.029	0.010
Inertia constant (s)	1.19	1.19	1.19	3
Resistances				
Stator resistance (pu)	0.003	0.003	0.003	0.003

References

- [1] D.K. Faddeev, V.N. Faddeyeva, Computing Methods of Linear Algebra. Moscow: Physical and Mathematical Literature Publishers, (in Russian), 1963, p. 655.
- [2] P. Kundur, G.J. Rogers, D.Y. Wong, L. Wang and M.G. Lauby, "A Comprehensive Computer program Package for Small Signal Stability Analysis of Large Power Systems," *IEEE Trans. on Power Systems*, vol. 3, pp. 635-642, Nov. 1990.
- [3] B. Hanzon, R.L.M. Peeters, A Faddeev Sequence Method of Solving Lyapunov and Sylvester Equations, *Linear Algebra and Its Appl.*, 1996, 241 -243, pp. 401 – 430.
- [4] I.B. Yadykin, "On Expansion of Gramians and Cross-Gramians of Continuous Control Systems Over Spectra of System Dynamics Matrices", in *Proc. 2011 18th IFAC World Congress.*, vol. 18, part 1, pp. 9313-9319.
- [5] S. K. Yee, J. V. Milanovic, F. M. Hughes, "Overview and Comparative Analysis of Gas Turbine Models for System Stability Studies," *IEEE Trans. on Power Systems*, vol. 23, No. 1, Feb. 2008, pp. 108-118.
- [6] A. Ahmetzyanov, A. Iskakov, A. Grigoryev, A. Matinyan, et al. "Gramians method of steady-state stability analysis in large electrical power systems," *Proc. 2012 Power Plant and Power Systems Control Symp.*, pp. 1-6.
- [7] N. Voropai, A. Domyshv, D. Efimov, D. Panasetsky, O. Sukhanov, I. Yadykin, U. Hager, T. Babnik, A. Grobovoy, A. Arestova, V. Fortunato, L. Bizumich, A. Sauhats, V. Strekovs, V. Chuvychin, D. Antonovs, N. Lisalek N, N. Balabin, X.P. Zhong, "Protection of Large Scale Power systems Interconnections," ICOEUR Deliverable 4.6. Tech. Rep. Available: <http://www.icoeur.eu>, D 4.6. pp. 1-84.